Asset pricing interpretations of the primary fiscal balance: A case of Japan

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Abstract: Chien et al. (2025) makes a controversial statement that the huge liabilities owed by the Japanese government are well backed by high β assets and claims, whose random payoffs evolve according to a net liability version of the primary fiscal balance (PFB). A major empirical ground for their statement is that the Japanese government reduced substantially its net liabilities by earning enormous capital gains from domestic and foreign risky assets from the mid-2010s through the mid-2020s. This paper theoretically demonstrates that if any asset is properly priced by a legitimate stochastic discount factor, then high β assets and claims held by the government improve the expected PFB, which is targeted by Chien et al., but they have no impact on the discounted PFB, which is of our major interest. Even if all parameters of the β-CAPM regression for excess returns on risky assets are considered to the fullest under constant discounting, the discounted PFB bears at most moderate β, and still yields negative risk premiums. With BoJ's net assets included, the results do not change at all. In conclusion, the present value of the future PFB is far short of the current valuation of the net liabilities.

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1. Introduction

Chien et al. (2025) makes a controversial statement that the Japanese government's net liabilities are well backed by high β assets and claims, whose random payoffs evolve according to a net liability version of the primary fiscal balance (hereafter, PFB). Consequently, its huge gross liabilities are still sustainable with such a high β claim on the fiscal surplus as well as a large amount of risky assets with high β, which are held by various bodies of the government. Their statement could justify gigantic public debt without resorting to the growth exceeding interest ($g > r$) argument as in Blanchard (2019) and others, or the violation of the transversality condition as in Saito (2021) and others. This paper examines their statement with extreme care, first theoretically by a standard asset pricing model, and second empirically by a standard β-CAPM regression.[2]

A major empirical ground for their statement is that the Japanese general government, consisting of the central and local governments, and the social security funds, successfully reduced its net liabilities by earning enormous amounts of capital gains from risky assets, both domestic and foreign, from the mid-2010s through the mid-2020s. According to the Flow of Funds Accounts compiled by the Bank of Japan (BoJ),[3] as shown in Figure 1-1, the gross liabilities (a red bar) expanded from the late 1990s to the early 2020s, but the net liabilities (a black line) ceased to increase in the early 2010s, and they started to decline from the late 2010s. As Figure 1-2 demonstrates, such a trend is more eminent in terms of relative to nominal GDP. A reason for the difference in a trend between gross liabilities and net ones is that most capital gains from risky assets are unrealized and are not included in the revenue of the conventional gross liability version of the PFB. On the other hand, such unrealized capital gains are reflected in the market valuation of gross assets, which are subtracted from gross liabilities in deriving net ones.

(insert Figure 1-1)

(insert Figure 1-2)

The government sector indeed allocated a significant portion of gross assets to domestic and foreign risky investment. Accordingly, the gross asset side moved quite differently from the gross liability side, depending on how equity returns, and exchange rates behaved in the form of unrealized returns. As shown in Figure 2-1, the central government, particularly the foreign

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[2] Huang and Litzenberger (1988) provide one of the most intensive and extensive discussions about β-CAPM.

[3] See Bank of Japan (1998-2024).

exchange fund special account, started to invest in foreign assets and domestic equities from the early 2000s, while as shown in Figure 2-2, the social security funds started to shift from the Japanese government bonds (JGBs) to domestic equities and foreign assets in the late 2000s. In the gross assets of the BoJ as another body of the integrated government, as shown in Figure 2-3, its investment shifted from treasury bills (short-term JGBs) to long-term JGBs, and domestic equities from the early 2010s. But BoJ's investment in foreign assets had been still negligible.

(insert Figure 2-1)

(insert Figure 2-2)

(insert Figure 2-3)

In this paper, we redefine the PFB according to not gross, but net liability dynamics. As in Jiang et al. (2024) and Chien et al. (2025), we treat such a net liability version of the PFB as a random payoff from the government's investment in risky assets, and its claim on the fiscal surplus, and discount it by a stochastic factor to derive the present value of the future PFB. Stepping back from a general setup by Jiang et al. (2024), we adopt as a concrete simple specification, a stochastic discount factor implicit in β-CAPM. Another important difference between Chien et al. and ours is that the former targets the expected PFB, but the latter focuses on the discounted PFB.

We challenge the above-mentioned controversial statement by Chien et al. in two respects. First, if a certain asset held by the government is priced properly by a legitimate stochastic discount factor, its high β improves the expected PFB, which is of their interest, but it has no impact on the discounted PFB, which is of our interest. Second, if the Euler equation does not hold with respect to excess returns, some or all parameters of the β-CAPM regression for risky excess returns may affect not only the expected PFB, but also the discounted PFB. However, even if such parameters are empirically considered to the fullest under constant discounting, the estimated discounted PFB bears at most moderate β, and still yields negative excess returns on the average. With BoJ's net assets included, these results do not change at all. In conclusion, the Japanese government's liabilities are not backed by risky assets as well as claims on the future fiscal surplus and are still unfunded despite active risky investment by various bodies of the integrated government.

This paper is organized as follows. In Section 2, we define the PFB according to the net liability dynamics and explore how parameters of the β-CAPM regression affect the present value of the future PFB. In Section 3, using the Flow of Funds Accounts, we compute the time series of a net liability version of the PFB, and estimate the present value of the future PFB. In Section 4, given our theoretical and empirical exercises, we conclude that the present value of the future PFB is far short of the current valuation of the net liabilities.

2. A net liability version of the primary fiscal balance

2.1. Constant discounting as risk neutrality

The primary fiscal balance (PFB) of the general government (GG) is usually determined according to the following gross liability dynamics.

$$\Delta D_t^{GG} = -[(T_t^{GG} + r_t^{GG} A_{t-1}^{GG}) - (G_t^{GG} + \Delta A_t^{GG})] + r_t^D D_{t-1}^{GG}, \tag{1}$$

where D_t^{GG} and A_t^{GG} are gross liabilities and gross assets outstanding at the end of time t. On the expenditure side, G_t^{GG} is defined as government expense, which excludes interest payment ($r_t^D D_{t-1}^{GG}$) and net asset purchase (ΔA_t^{GG}). On the other hand, the revenue side consists of tax revenue (T_t^{GG}) and returns from gross assets ($r_t^{GG} A_{t-1}^{GG}$). r_t^D and r_t^{GG} denote nominal interest rates on short-term JGBs and nominal yields on gross assets. Thus, the PFB is defined as follows.

$$PFB_t^{GL} = (T_t^{GG} + r_t^{GG} A_{t-1}^{GG}) - (G_t^{GG} + \Delta A_t^{GG}) \tag{2}$$

There are two potential problems in using the above gross liability version of the PFB. First, returns from gross assets record only realized returns ($r_t^{GG,R} A_{t-1}^{GG}$), and do not include any unrealized capital gain or loss ($r_t^{GG,UR} A_{t-1}^{GG}$). Second, net asset purchase ($\Delta A_t^{GG,R}$) is recorded only on a purchase and sale basis, and it does not reflect any market valuation.

This paper instead proposes a *net* liability version of the PFB with due consideration for unrealized returns from gross assets ($r_t^{GG,UR} A_{t-1}^{GG}$), as well as the market valuation for net liabilities ($D_t^{GG} - A_t^{GG}$) and its increment ($\Delta(D_t^{GG} - A_t^{GG})$), in which both A_t^{GG} and A_{t-1}^{GG} are evaluated in terms of market prices. That is, the PFB is determined according to the following net liability dynamics.

$$\Delta(D_t^{GG} - A_t^{GG}) = -\left[(T_t^{GG} - G_t^{GG}) + \left(r_t^{GG,R} + r_t^{GG,UR} - r_t^D\right) A_{t-1}^{GG}\right] + r_t^D (D_{t-1}^{GG} - A_{t-1}^{GG}) \tag{3}$$

Then, a net liability version of the PFB is now defined as

$$PFB_t^{NL} = (T_t^{GG} - G_t^{GG}) + \left(r_t^{GG,R} + r_t^{GG,UR} - r_t^D\right)A_{t-1}^{GG} \quad (4)$$

A comparison between (2) and (4) suggests that in the latter, net asset purchase (ΔA_t^{GG}) is dropped, but unrealized excess returns ($\left(r_t^{GG,UR} - r_t^D\right)A_{t-1}^{GG}$) are included. One important feature of PFB_t^{NL} is that not gross returns ($r_t^{GG,R} + r_t^{GG,UR}$), which appear in Chien et al.'s net liability dynamics,[4] but excess returns ($r_t^{GG,R} + r_t^{GG,UR} - r_t^D$) show up in the right-hand side of equation (3) or (4). As shown below, it is for this feature that we are interested in not the expected PFB, but the discounted PFB.

Let us assume that agents are risk neutral, and that they discount future payoffs by a constant factor. If the future PFB is discounted by a constant interest rate on short-term JGBs (r^D), then equation (3) is rewritten in a forward-looking manner.

$$D_{t-1}^{GG} - A_{t-1}^{GG} = E_{t-1}\left\{\frac{\left[(T_t^{GG} - G_t^{GG}) + \left(r_t^{GG,R} + r_t^{GG,UR} - r^D\right)A_{t-1}^{GG}\right] + (D_t^{GG} - A_t^{GG})}{1 + r^D}\right\}$$

Here, both sides of the above equation are divided by one-period lagged gross liabilities D_{t-1}^{GG}. Then, it is further developed in a sequential manner.

$$1 - \frac{A_{t-1}^{GG}}{D_{t-1}^{GG}} = \sum_{\tau=0}^{\infty} \mathrm{E}_{t-1}\left\{\frac{1}{\left(1+r^D\right)^{\tau+1}}\frac{D_{t-1+\tau}^{GG}}{D_{t-1}^{GG}}\left[\frac{\left(T_{t+\tau}^{GG}-G_{t+\tau}^{GG}\right)+\left(r_t^{GG,R}+r_t^{GG,UR}-r^D\right)A_{t-1+\tau}^{GG}}{D_{t-1+\tau}^{GG}}\right]\right\} + \frac{1}{D_{t-1}^{GG}}\lim_{\tau\to\infty}\mathrm{E}_t\left[\frac{D_{t+\tau}^{GG}-A_{t+\tau}^{GG}}{\left(1+r^D\right)^{\tau+1}}\right]. \quad (5)$$

Let us assume for simplification that (i) $\frac{D_t^{GG}}{D_{t-1}^{GG}}$ moves around one at a steady state, and $\frac{D_{t-1+\tau}^{GG}}{D_{t-1}^{GG}} = 1 + \sum_{i=0}^{\tau-1}\varepsilon_{t+i}$ with white noise ε_{t+i}, and (ii) $\lim_{\tau\to\infty}\mathrm{E}_t\left[\frac{D_{t+\tau}^{GG}-A_{t+\tau}^{GG}}{\left(1+r^D\right)^{\tau+1}}\right] = 0$ by a transversality condition. By assumption (ii), we suppose that the current valuation of the net liabilities is equal to the present value of the future PFB.

Then, equation (5) is further rewritten as

$$1 - \frac{A_{t-1}^{GG}}{D_{t-1}^{GG}} = \sum_{\tau=0}^{\infty} \mathrm{E}_{t-1}\left\{\frac{1}{\left(1+r^D\right)^{\tau+1}}\frac{D_{t-1+\tau}^{GG}}{D_{t-1}^{GG}}\mathrm{E}_{t-1+\tau}\left[\left(\frac{T_{t+\tau}^{GG}-G_{t+\tau}^{GG}}{D_{t-1+\tau}^{GG}}\right) + \left(r_{t+\tau}^{GG,R} + r_{t+\tau}^{GG,UR} - r^D\right)\frac{A_{t-1+\tau}^{GG}}{D_{t-1+\tau}^{GG}}\right]\right\}$$

[4] Chien et al. (2025) formulate as the net liability dynamics, $D_t - A_t = (G_t - T_t) + (1 + r_t^D)D_{t-1} - (1 + r_t^R + r_t^{UR})A_{t-1}$.

$$= \sum_{\tau=0}^{\infty}\left\{\frac{1}{(1+r^D)^{\tau+1}}\left[\mathrm{E}_{t-1}\left(\frac{T_{t+\tau}^{GG}-G_{t+\tau}^{GG}}{D_{t-1+\tau}^{GG}}\right)+\mathrm{E}_{t-1}\left[\frac{A_{t-1+\tau}^{GG}}{D_{t-1+\tau}^{GG}}\mathrm{E}_{t-1+\tau}\left(r_{t+\tau}^{GG,R}+r_{t+\tau}^{GG,UR}-r^D\right)\right]\right]\right\}. \quad (6)$$

Taking the unconditional expectation for both sides, the present value of PFB_t^{NL} relative to D_{t-1}^{GG} leads to

$$1-\mathrm{E}\left(\frac{A_{-1}^{GG}}{D_{-1}^{GG}}\right)=\frac{1}{r^D}\left\{\mathrm{E}\left(\frac{T^{GG}-G^{GG}}{D_{-1}^{GG}}\right)+\mathrm{E}\left[\frac{A_{-1}^{GG}}{D_{-1}^{GG}}\left(r^{GG,R}+r^{GG,UR}-r^D\right)\right]\right\}.$$

In this way, the unconditional expectation of PFB_t^{NL} relative to D_{t-1}^{GG} corresponds to

$$\mathrm{E}\left(\frac{PFB^{NL}}{D_{-1}^{GG}}\right)=\mathrm{E}\left(\frac{T^{GG}-G^{GG}}{D_{-1}^{GG}}\right)+\mathrm{E}\left[\frac{A_{-1}^{GG}}{D_{-1}^{GG}}\left(r^{GG,R}+r^{GG,UR}-r^D\right)\right]. \quad (7)$$

However, taking risk neutrality seriously as the underlying assumption, average excess returns or risk premia should be equal to zero. That is, $\mathrm{E}_{t-1+\tau}\left(r_{t+\tau}^{GG,R}+r_{t+\tau}^{GG,UR}-r^D\right)$ in the right-hand side of equation (6) degenerates to zero. Accordingly, the unconditional expectation of PFB_t^{NL} relative to D_{t-1}^{GG} turns out to be $\mathrm{E}\left(\frac{T^{GG}-G^{GG}}{D_{-1}^{GG}}\right)$, and its present value relative to D_{t-1}^{GG} reduced to

$$1-\mathrm{E}\left(\frac{A_{-1}^{GG}}{D_{-1}^{GG}}\right)=\frac{1}{r^D}\mathrm{E}\left(\frac{T^{GG}-G^{GG}}{D_{-1}^{GG}}\right). \quad (8)$$

Under constant discounting as risk neutrality, the discounted PFB_t^{NL} as well as its present value $\left(1-\mathrm{E}\left(\frac{A_{-1}^{GG}}{D_{-1}^{GG}}\right)\right)$ have nothing to do with how high excess returns are from GG's risky investment. The only source of positive cash flows from gross liabilities is the average surplus in the conventional PFB, or $\mathrm{E}\left(T^{GG}-G^{GG}\right)>0$.

2.2. Stochastic discounting as risk aversion

We below consider risk aversion instead of risk neutrality to evaluate possible impacts from excess returns from GG's risky investment on the present value of PFB_t^{NL}. We adopt a time-varying discount factor, which discounts future positive (negative) payoffs more (less) heavily.

Concretely, the following stochastic discount factor implicit in β-CAPM is chosen for this purpose.

$$DF_t = \frac{1 - \frac{\mathrm{E}(r^m - r^D)}{\mathrm{E}(r^m - r^D)^2}(r_t^m - r^D)}{1 + r^D}, \tag{9}$$

where $r_t^m - r^D$ is an excess market return, and it is assumed to follow

$$r_t^m - r^D = \mathrm{E}(r^m - r^D) + \epsilon_t^m \tag{10}$$

with ϵ_t^m white noise. Under this simplifying assumption, only ϵ_t^m represents the aggregate market risk.

The above stochastic discount factor DF_t is linear in a random payoff $r_t^m - r^D$. In addition, it is negatively corelated with any excess return $r_t^i - r^D$; that is, it discounts positive (negative) realization of excess returns more (less) heavily.

By construction, DF_t is orthogonal to excess market returns $r_t^m - r^D$ on the average.

$$\mathrm{E}_{t-1}[DF_t(r_t^m - r^D)] = \mathrm{E}_{t-1}\left[\frac{1 - \frac{\mathrm{E}(r^m - r^D)}{\mathrm{E}(r^m - r^D)^2}(r_t^m - r^D)}{1 + r^D}(r_t^m - r^D)\right] = 0 \tag{11}$$

As a legitimate discount factor, DF_t should be orthogonal to any other excess return on asset i $(r_t^i - r^D)$ on the average.

$$\mathrm{E}_{t-1}\left[DF_t(r_t^i - r^D)\right] = \mathrm{E}_{t-1}\left[\frac{1 - \frac{\mathrm{E}(r^m - r^D)}{\mathrm{E}(r^m - r^D)^2}(r_t^m - r^D)}{1 + r^D}(r_t^i - r^D)\right] = 0 \qquad \forall\, i. \tag{12}$$

Using $\mathrm{E}_{t-1}(x_t y_t) = \mathrm{Cov}_{t-1}(x_t, y_t) + \mathrm{E}_{t-1}(x_t)\mathrm{E}_{t-1}(y_t)$, we can derive β-CAPM from equations (10), (11), and (12) as follows.

$$r_t^i - r^D = \beta^i(r_t^m - r^D) + \eta_t^i, \tag{13}$$

where $\beta^i = \frac{Cov(r_t^i - r^D, r_t^m - r^D)}{Var(r_t^m - r^D)}$, and η_t^i is white noise. Note that a constant term is absent in the right-hand side of equation (13).

Let us apply $DF_t = \frac{1 - \frac{\mathrm{E}(r^m - r^D)}{\mathrm{E}(r^m - r^D)^2}(r_t^m - r^D)}{1 + r^D}$ instead of $\frac{1}{1+r^D}$ to discount the future PFB, which is defined by equation (4).

$$1 - \frac{A_{t-1}^{GG}}{D_{t-1}^{GG}} = \sum_{\tau=0}^{\infty} \mathrm{E}_{t-1}\left\{\prod_{i=0}^{\tau}(DF_{t+i})\,\frac{D_{t-1+\tau}^{GG}}{D_{t-1}^{GG}}\,\mathrm{E}_{t+\tau-1}\left[\frac{(T_{t+\tau}^{GG} - G_{t+\tau}^{GG}) + \left(r_{t+\tau}^{GG,R} + r_{t+\tau}^{GG,UR} - r_{t+\tau}^{D}\right)A_{t-1+\tau}^{GG}}{D_{t-1+\tau}^{GG}}\right]\right\}$$

$$= \sum_{\tau=0}^{\infty} \mathrm{E}_{t-1}\left\{\frac{D_{t-1+\tau}^{GG}}{D_{t-1}^{GG}}\prod_{i=0}^{\tau-1}(DF_{t+i})\,\mathrm{E}_{t-1+\tau}\left[DF_{t+\tau}\frac{(T_{t+\tau}^{GG} - G_{t+\tau}^{GG}) + \left(r_{t+\tau}^{GG,R} + r_{t+\tau}^{GG,UR} - r_{t+\tau}^{D}\right)A_{t-1+\tau}^{GG}}{D_{t-1+\tau}^{GG}}\right]\right\}.$$

Using equation (13), $r_{t+\tau}^{GG,R} + r_{t+\tau}^{GG,UR} - r_{t+\tau}^{D}$ is replaced with $\beta^{GG}(r_{t+\tau}^{m} - r^D) + \eta_{t+\tau}^{GG}$.

$$1 - \frac{A_{t-1}^{GG}}{D_{t-1}^{GG}} = \sum_{\tau=0}^{\infty} \mathrm{E}_{t-1}\left\{\frac{D_{t-1+\tau}^{GG}}{D_{t-1}^{GG}}\prod_{i=0}^{\tau-1}(DF_{t+i})\,\mathrm{E}_{t-1+\tau}\left[DF_{t+\tau}\frac{(T_{t+\tau}^{GG} - G_{t+\tau}^{GG}) + (\beta^{GG}(r_{t+\tau}^{m} - r^D) + \eta_{t+\tau}^{GG})A_{t-1+\tau}^{GG}}{D_{t-1+\tau}^{GG}}\right]\right\} \quad (14)$$

Noting that $\mathrm{E}_{t+\tau-1}[DF_{t+\tau}(\beta^{GG}(r_{t+\tau}^{m} - r^D) + \eta_{t+\tau}^{GG})] = 0$ by equation (11), we simplify equation (14) as

$$1 - \frac{A_{t-1}^{GG}}{D_{t-1}^{GG}} = \sum_{\tau=0}^{\infty} \mathrm{E}_{t-1}\left\{\prod_{i=0}^{\tau-1}(DF_{t+i})\mathrm{E}_{t-1+\tau}\left[DF_{t+\tau}\frac{T_{t+\tau}^{GG} - G_{t+\tau}^{GG}}{D_{t-1+\tau}^{GG}}\right]\right\}. \quad (15)$$

A fundamental reason for the absence of excess returns in equation (15) is that the past realization of capital gains and losses have already been reflected in the evaluation of the current gross assets A_{t-1}^{GG}. In terms of the future possibilities of capital gains and losses, however, positive (negative) excess returns are discounted more (less) heavily under stochastic discounting, and the discounted excess returns degenerate to zero. Accordingly, $\beta^{GG}(r_{t+\tau}^{m} - r^D)$ has no impact on the discounted PFB.

Regressing $\frac{T_{t+\tau}^{GG} - G_{t+\tau}^{GG}}{D_{t-1+\tau}^{GG}}$ on $r_t^m - r^D$, defined by equation (10), leads to

$$\frac{T_{t+\tau}^{GG} - G_{t+\tau}^{GG}}{D_{t-1+\tau}^{GG}} = \alpha^{T-G} + \beta^{T-G}(r_{t+\tau}^{m} - r^D) + \eta_t^{T-G}, \quad (16)$$

where $\beta^{T-G} = \frac{Cov\left(\frac{T^{GG} - G^{GG}}{D_{-1}^{GG}}, r^m - r^D\right)}{\mathrm{Var}(r^m - r^D)}$, and $\alpha^{T-G} = \mathrm{E}\left(\frac{T^{GG} - G^{GG}}{D_{-1}^{GG}}\right) - \beta^{T-G}E(r^m - r^D)$.

Replacing $\frac{T_{t+\tau}^{GG} - G_{t+\tau}^{GG}}{D_{t-1+\tau}^{GG}}$ by $\alpha^{T-G} + \beta^{T-G}(r_{t+\tau}^{m} - r^D) + \eta_t^{T-G}$, we obtain

$$\mathrm{E}_{t-1+\tau}\left[DF_{t+\tau}\frac{T_{t+\tau}^{GG}-G_{t+\tau}^{GG}}{D_{t-1+\tau}^{GG}}\right]=\mathrm{E}_{t-1+\tau}\{DF_{t+\tau}[\alpha^{T-G}+\beta^{T-G}(r_{t+\tau}^{m}-r^{D})+\eta_{t}^{T-G}]\}$$

$$=\mathrm{E}_{t-1+\tau}(DF_{t+\tau})\left[\mathrm{E}\left(\frac{T^{GG}-G^{GG}}{D_{-1}^{GG}}\right)-\beta^{T-G}E(r^{m}-r^{D})\right],$$

again thanks to $\mathrm{E}_{t+\tau-1}[DF_{t+\tau}(\beta^{GG}(r_{t+\tau}^{m}-r^{D})+\eta_{t+\tau}^{GG})]=0$ by equation (11), and by $\alpha^{T-G}=\mathrm{E}\left(\frac{T^{GG}-G^{GG}}{D_{-1}^{GG}}\right)-\beta^{T-G}E(r^{m}-r^{D})$.

Applying equation (10), and using $\mathrm{E}(DF)=\left\{1-\frac{[\mathrm{E}(r^{m}-r^{D})]^{2}}{\mathrm{E}(r^{m}-r^{D})^{2}}\right\}/(1+r^{D})\approx 1/\left\{1+r^{D}+\frac{[\mathrm{E}(r^{m}-r^{D})]^{2}}{\mathrm{E}(r^{m}-r^{D})^{2}}\right\}$ when $\mathrm{E}(r^{m}-r^{D})^{2}$ is close to $\mathrm{Var}(r^{m}-r^{D})$, equation (15) is further simplified as

$$1-\mathrm{E}\left(\frac{A_{-1}^{GG}}{D_{-1}^{GG}}\right)=\sum_{\tau=0}^{\infty}\left[\left\{1-\frac{[\mathrm{E}(r^{m}-r^{D})]^{2}}{\mathrm{E}(r^{m}-r^{D})^{2}}\right\}/(1+r^{D})\right]^{\tau+1}\left[\mathrm{E}\left(\frac{T^{GG}-G^{GG}}{D_{-1}^{GG}}\right)-\beta^{T-G}E(r^{m}-r^{D})\right]$$

$$\approx\frac{1}{r^{D}+\frac{[\mathrm{E}(r^{m}-r^{D})]^{2}}{\mathrm{E}(r^{m}-r^{D})^{2}}}\left[\mathrm{E}\left(\frac{T^{GG}-G^{GG}}{D_{-1}^{GG}}\right)-\beta^{T-G}E(r^{m}-r^{D})\right]. \qquad (17)$$

As equation (17) implies, the present value of PFB_{t}^{NL} relative to D_{t-1}^{GG} has nothing to do with how high excess returns are from GG's risky investment. It is even similar to equation (8), and the major source of positive cash flows from gross liabilities is again $\mathrm{E}(T^{GG}-G^{GG})>0$.

But there are two differences between constant discounting and stochastic discounting. First, higher constant discount rates are applied in the latter ($r^{D}+\frac{[\mathrm{E}(r^{m}-r^{D})]^{2}}{\mathrm{E}(r^{m}-r^{D})^{2}}>r^{D}$). Second, if β^{T-G} is negative, or $\frac{T_{t}^{GG}-G_{t}^{GG}}{D_{t-1}^{GG}}$ is negatively correlated with $r_{t}^{m}-r^{D}$, then the discounted cash flows from net liabilities improve. This consequence is that counter-cyclical $T_{t}^{GG}-G_{t}^{GG}$ has an insurance effect on the discounted cash flows.

Restating the above implication, GG's life-time budget constraint (17) is independent of how the GG allocates funds among various risky assets. What does matter directly for GG's budget constraint is not the future possibility of capital gains and losses, but the past realization of capital gains and losses, which is reflected in the current valuation of the gross assets, and is deducted from the gross liabilities.

2.3. A case of the failure of the Euler equations with respect to excess returns

As demonstrated in the previous subsection, as a consequence of strong restrictions from asset pricing theory, realized and unrealized returns from risky investment ($r_t^{GG,R} + r_t^{GG,UR}$) disappear in a transition from equation (6) or (7) to equation (8) under constant discounting, and from equation (14) to equation (15) or (17) under stochastic discounting. In principle, such risky returns have no impact on the discounted PFB or its present value.

However, it is well-known that the orthogonality condition between stochastic discount factors and excess returns, which is represented by the Euler equation (12), often breaks down due to various transaction constraints. In particular, excess returns tend to be too high on the average given a certain stochastic factor. Consequently, short in risk-free assets and long in risky assets opens arbitrage opportunities for investors who can hold such risky assets. In the current cases, the Euler relationship holds in not equality, but inequality as follows.

$$\frac{1}{1+r^D}\mathrm{E}_{t-1+\tau}\left(r_{t+\tau}^{GG,R} + r_{t+\tau}^{GG,UR} - r^D\right) = c_t^{GG} > 0$$

$$\mathrm{E}_{t-1+\tau}\left[\frac{1-\frac{\mathrm{E}\left(r^m-r^D\right)}{\mathrm{E}\left(r^m-r^D\right)^2}\left(r_{t+\tau}^m-r^D\right)}{1+r^D}\frac{\left(r_{t+\tau}^{GG,R}+r_{t+\tau}^{GG,UR}-r_{t+\tau}^D\right)A_{t-1+\tau}^{GG}}{D_{t-1+\tau}^{GG}}\right] = c_t^{GG} > 0$$

Suppose that equation (11) holds by construction, but equation (12) fails to hold for some transaction constraints as follows.

$$\mathrm{E}_{t-1}\left[\frac{1-\frac{\mathrm{E}\left(r^m-r^D\right)}{\mathrm{E}\left(r^m-r^D\right)^2}\left(r_t^m-r^D\right)}{1+r^D}\left(r_t^i - r^D\right)\right] = c^i > 0 \qquad (18)$$

Note that c^i is constant over time as a simplifying assumption.

Together with equation (10), β-CAPM is modified from equation (13) to that with a positive constant term.

$$r_t^i - r^D = \alpha^i + \beta^i\left(r_t^m - r^D\right) + \eta_t^i, \qquad (19)$$

where $\alpha^i = c^i E[r^m - r^D] > 0$,[5] and $\beta^i = \frac{Cov(r_t^i - r^D, r_t^m - r^D)}{Var(r_t^m - r^D)}$.

If α^i in equation (19) is significantly positive in the β-CAPM regression, then investment in asset i creates arbitrage opportunities for those who can have long positions in this asset. For example, borrowing at r^D, investing loaned money in asset i, and immediately selling it at futures results in a budget-free profit for them.

Using equations (16) and (19), $\frac{(T_{t+\tau}^{GG} - G_{t+\tau}^{GG}) + \left(r_{t+\tau}^{GG,R} + r_{t+\tau}^{GG,UR} - r_{t+\tau}^{D}\right) A_{t-1+\tau}^{GG}}{D_{t-1+\tau}^{GG}}$ can be replaced by $\frac{\left[\alpha^{T-G} + \beta^{T-G}(r_{t+\tau}^m - r^D) + \eta_t^{T-G}\right] + \left[\alpha^{GG} + \beta^{GG}(r_{t+\tau}^m - r^D) + \eta_{t+\tau}^{GG}\right] A_{t-1+\tau}^{GG}}{D_{t-1+\tau}^{GG}}$. We discount this time $t+\tau$ payoff as of time $t-1+\tau$ by two types of discount factors. First, applying the same stochastic factor implicit in β-CAPM, we obtain as follows.

$$\mathrm{E}_{t+\tau-1}\left\{\frac{1 - \frac{\mathrm{E}\left(r^m - r^D\right)}{\mathrm{E}\left(r^m - r^D\right)^2}(r_t^m - r^D)}{1+r^D}\left[\left[\alpha^{T-G} + \beta^{T-G}(r_{t+\tau}^m - r^D) + \eta_t^{T-G}\right] + \frac{\left[\alpha^{GG} + \beta^{GG}(r_{t+\tau}^m - r^D) + \eta_{t+\tau}^{GG}\right] A_{t-1+\tau}^{GG}}{D_{t-1+\tau}^{GG}}\right]\right\}$$

$$= \frac{1 - \frac{\left[\mathrm{E}\left(r^m - r^D\right)\right]^2}{\mathrm{E}\left(r^m - r^D\right)^2}}{1+r^D}\left[\mathrm{E}\left(\frac{T^{GG} - G^{GG}}{D_{-1}^{GG}}\right) - \beta^{T-G} E(r^m - r^D) + \alpha^{GG}\frac{A_{t-1+\tau}^{GG}}{D_{t-1+\tau}^{GG}}\right], \quad (20)$$

Note that $r_{t+\tau}^m - r^D$ is orthogonal to $\left[1 - \frac{\mathrm{E}(r^m - r^D)}{\mathrm{E}\left(r^m - r^D\right)^2}(r_t^m - r^D)\right] / (1 + r^D)$ on the average. In equation (20), β^{GG} again disappears, but α^{GG} shows up.

Second, adopting a constant discount factor, we derive the following.

$$\frac{1}{1+r^D}\mathrm{E}_{t+\tau-1}\left[\frac{(T_{t+\tau}^{GG} - G_{t+\tau}^{GG}) + (\alpha^{GG} + \beta^{GG}(r_{t+\tau}^m - r^D) + \eta_{t+\tau}^{GG}) A_{t-1+\tau}^{GG}}{D_{t-1+\tau}^{GG}}\right]$$

$$= \frac{1}{1+r^D}\left[\left(\alpha^{T-G} + \beta^{T-G}\mathrm{E}(r^m - r^D)\right) + \left(\alpha^{GG} + \beta^{GG}\mathrm{E}(r^m - r^D)\right)\frac{A_{t-1+\tau}^{GG}}{D_{t-1+\tau}^{GG}}\right], \quad (21)$$

where $\mathrm{E}\left(\frac{T^{GG} - G^{GG}}{D_{-1}^{GG}}\right) = \alpha^{T-G} + \beta^{T-G}\mathrm{E}\left(r^m - r^D\right)$.

All αs and βs of equations (16) and (19) appear in equation (21). That is, all parameters of

[5] Note that α^i in equation (19) is different from $\alpha^i = \frac{\mathrm{Var}_t(r_t^i - r^d)}{\mathrm{E}_t(r_t^i - r^d)}$ in Chien et al. (2025).

the β-CAPM regression show up for not only the expected PFB, but also the discounted PFB. The sign in front of β^{T-G} is not negative, but positive, because the insurance effect is not present any more under constant discounting.

Rigorously, we have to admit that applying β-CAPM to evaluate the expected PFB, and applying constant factors to discount it are incompatible with each other. Thus, the second method is rather ad-hoc. Nevertheless, we still consider it because equation (21) reflects the impact of parameters of the β-CAPM regression to the fullest.

In the first case, the present value of the relative PFB is modified from equation (17) to

$$1-\mathrm{E}\left(\frac{A_{-1}^{GG}}{D_{-1}^{GG}}\right)\approx\frac{1}{r^D+\frac{\left[\mathrm{E}\left(r^m-r^D\right)\right]^2}{\mathrm{E}\left(r^m-r^D\right)^2}}\left[\mathrm{E}\left(\frac{T^{GG}-G^{GG}}{D_{-1}^{GG}}\right)-\beta^{T-G}E(r^m-r^D)+\alpha^{GG}\frac{A_{-1}^{GG}}{D_{-1}^{GG}}\right], \tag{22}$$

while in the second case, on the other hand, it modified from equation (8) to

$$1-\mathrm{E}\left(\frac{A_{-1}^{GG}}{D_{-1}^{GG}}\right)=\frac{1}{r^D}\left[\left(\alpha^{T-G}+\beta^{T-G}\mathrm{E}(r^m-r^D)\right)+\left(\alpha^{GG}+\beta^{GG}\mathrm{E}(r^m-r^D)\right)\mathrm{E}\left(\frac{A^{GG}}{D^{GG}}\right)\right]. \tag{23}$$

Unlike in equations (17) and (8), some or all parameters from the β-CAPM regression appear in equations (22) and (23). That is, GG's behavior in risky investment finally appears in the evaluation of the present value of the future PFB. In equation (22), how the GG exploits arbitrage opportunities with positive α^{GG} improves the present value of the PFB. In equation (23), though it is quite ad-hoc, how the GG holds risky assets with high β^{GG} enhances it.

In equation (23), the effects of αs and βs on the discounted PFB is equivalent to those on the expected PFB. In this regard, equation (23) is important in comparison between Chien et al. and our paper. Chien et al. consider possible effects from the β-CAPM regression at the level of not the discounted PFB, but the expected PFB.

More concretely, all αs are set at zero in Chien et al. Our β^{T-G} corresponds to their $\frac{D_t-A_t}{D_t}\beta_t^S$, where β_t^S is β of the conventional surplus $\frac{T_t-G_t}{D_t-A_t}$, while our $\frac{A_t}{D_t}\beta^{GG}$ corresponds to their $\frac{A_t}{D_t}\beta_t^A$. According to their calibration (not estimation), both βs of the surplus claim and the risky assets are quite high. Given $\frac{A_t}{D_t}=0.66$, $\beta_t^S=0.45$, and $\beta_t^A=0.5$, β of the gross liability is equal to $(1-0.66)\times 0.45+0.66\times 0.5=0.48$. One of our goals is to compare this number 0.48 with our estimate of $\beta^{T-G}\mathrm{E}(r^m-r^D)+\frac{A^{GG}}{D^{GG}}\beta^{GG}\mathrm{E}(r^m-r^D)$ in equation (23).

Let us summarize possible implications from this section for the recent Japanese fiscal situation. A net liability version of the PFB, defined by equation (4), differs from its conventional gross liability version, defined by equation (2), by including unrealized returns from risky assets. However, applying either constant discounting or stochastic discounting to this net liability version of the PFB, the present value of the future PFB (equations (8) and (17)) does not depend on how much risk the government is taking for financial investment.

Consequently, in either discounting, the present value of a net liability version of the PFB is even closer to that of its gross liability version (equation (2)), which has been chronically negative in Japan. In terms of rigorous asset pricing implications, the fact that various bodies of the GG started to invest in risky assets from the twenty first century does not matter at all for the evaluation of the present value of the future PFB.

However, such theoretical restrictions often fail to hold in the real world. In particular, the Euler equation with respect to excess returns may not be applied to actual asset pricing data. Once the deviation of the Euler equation is taken into consideration, how much risk the GG is taking is still significant in evaluating the discounted PFB. More concretely, as equations (22) and (23) implies, some or all parameters from the β-CAPM regression parameters do matter for evaluating not only the expected PFB, but also the discounted PFB. Accordingly, we can organize a sort of forum where Chien et al.'s calibration is comparable to our estimation. In the next section, we empirically explore whether the degree to which the government is making risky investment helps have the discounted PFB reversed from negative to positive

3. Construction of a net liability version of the PFB from the Flow of Funds Accounts

3.1. Data sources

To compute the PFB, we mainly use the quarterly Flow of Funds Accounts, compiled by the BoJ. These accounts consist of the stock tables (financial assets and liabilities), the flow tables (financial transactions), and the reconciliation tables (reconciliation between flows and stocks) from the first quarter of 1998 up to the fourth quarter of 2024. In addition, we draw the series of interest paid by the GG from the annual report of the National Accounts[6] from the first quarter of 1998 up to the first quarter of 2024.

An increment in net liabilities of the GG ($\Delta(D_t^{GG} - A_t^{GG})$) can be computed by a first-difference between $D_t^{GG} - A_t^{GG}$ and $D_{t-1}^{GG} - A_{t-1}^{GG}$ from the stock table. Out of $\Delta(D_t^{GG} - A_t^{GG})$, the realized components ($\Delta(D_t^{GG} - A_t^{GG})^R$) correspond to the financial surplus/deficit of the flow table, while the unrealized components ($\Delta(D_t^{GG} - A_t^{GG})^{UR}$) correspond to that of the reconciliation table.

[6] See Economic and Social Research (2024).

Then, $\Delta(D_t^{GG} - A_t^{GG}) = \Delta(D_t^{GG} - A_t^{GG})^R + \Delta(D_t^{GG} - A_t^{GG})^{UR}$ holds.

There are clear seasonal patterns in the realized components ($\Delta(D_t^{GG} - A_t^{GG})^R$). Thus, we take one-year moving averages of $(\Delta(D_t - A_t)^R$ for not only the GG, but also the central and local governments (denoted by CLGs) and the social security funds (denoted by SSFs). Thank to adopting one-year moving average instead of seasonal adjustment, $\Delta(D_t^{GG} - A_t^{GG})^R = \Delta(D_t^{CLGs} - A_t^{CLGs})^R + \Delta(D_t^{SSFs} - A_t^{SSFs})^R$ holds exactly, but the series starts from not the first quarter of 1998, but its fourth quarter, and still ends at the fourth quarter of 2024. Accordingly, the sample period is between the fourth quarter of 1998 and the fourth quarter of 2024, which is almost the same as that of Chien et al.'s calibration.

The series of interest paid by the GG ($r_r^D D_t^{GG}$) ends at the first quarter of 2024. Thus, the same numbers as the second quarter through the fourth of 2023 are inserted into the corresponding quarters of 2024. Because there are also seasonal patterns in $r_r^D D_t^{GG}$, one-year moving average is taken for the series. The sample period is then the same as above.

Opportunity costs to hold gross assets A_{t-1}^{GG} is computed by $r_t^D A_{t-1}^{GG} = (r_r^D D_t^{GG}) \frac{A_{t-1}^{GG}}{D_{t-1}^{GG}}$, where $\frac{A_{t-1}^{GG}}{D_{t-1}^{GG}}$ is available from the stock table. Similarly, $r_t^D A_{t-1}^{CLGs} = (r_r^D D_t^{GG}) \frac{A_{t-1}^{CLGs}}{D_{t-1}^{GG}}$, and $r_t^D A_{t-1}^{SSFs} = (r_r^D D_t^{GG}) \frac{A_{t-1}^{SSFs}}{D_{t-1}^{GG}}$.

3.2. Observed values of several versions of the PFB

We can now compute gross and net liability versions of the PFB. From equation (1), we derive a gross liability version of the PFB.

$$PFB_t^{GG,gross} = (T_t^{GG} - G_t^{GG}) + r_t^{GG,R} A_{t-1}^{GG} - \Delta A_t^{GG,R} = -\Delta D_t^{GG} + r_t^D D_{t-1}^{GG}, \tag{1'}$$

where ΔD_t^{GG} can be computed from the stock table, and $r_t^D D_{t-1}^{GG}$ is available from the National Accounts.

From equation (3), we derive a net liability version of the PFB.

$$PFB_t^{GG,net} = (T_t^{GG} - G_t^{GG}) + \left(r_t^{GG,R} + r_t^{GG,UR} - r_t^D\right) A_{t-1}^{GG} = -\Delta(D_t^{GG} - A_t^{GG}) + r_t^D D_{t-1}^{GG} - r_t^D A_{t-1}^{GG} \tag{3'}$$

Equation (3') can be decomposed into realized and unrealized components.

$$PFB_t^{GG,net,R} = (T_t^{GG} - G_t^{GG}) + r_t^{GG,R} A_{t-1}^{GG} = -\Delta(D_t^{GG} - A_t^{GG})^R + r_t^D D_{t-1}^{GG} \tag{3'-1}$$

$$PFB_t^{GG,net,UR} = \left(r_t^{GG,UR} - r_t^D\right)A_{t-1}^{GG} = -\Delta(D_t^{GG} - A_t^{GG})^{UR} - r_t^D A_{t-1}^{GG} \quad (3\text{'-}2)$$

Equation (3'-2) is further divided into CLGs' assets and SSFs' assets.

$$PFB_t^{CLGs,UR} = \left(r_t^{CLGs,UR} - r_t^D\right)A_{t-1}^{CLGs} = -\Delta(D_t^{CLGs} - A_t^{CLGs})^{UR} - r_t^D A_{t-1}^{CLGs} \quad (3\text{'-}2\text{-}1)$$

$$PFB_t^{SSFs,UR} = \left(r_t^{SSFs,UR} - r_t^D\right)A_{t-1}^{SSFs} = -\Delta(D_t^{SSFs} - A_t^{SSFs})^{UR} - r_t^D A_{t-1}^{SSFs} \quad (3\text{'-}2\text{-}2)$$

In addition, realized and unrealized returns from net assets for the BoJ as another body of the integrated government (denoted by IG) is determined as follows.

$$R_t^{BoJ,net} = \Delta\left(A_t^{BoJ} - D_t^{BoJ}\right) = \Delta\left(A_t^{BoJ} - D_t^{BoJ}\right)^R + \Delta\left(A_t^{BoJ} - D_t^{BoJ}\right)^{UR} \quad (24)$$

The BoJ never paid any interest on required and excess reserves up to October 2008. From then, it started to pay interest rates equal to only 0.1% or less on excess reserves. BoJ's gross liabilities occupy around 95% of its gross assets. Thus, $r_t^{D,BoJ}A_{t-1}^{BoJ} - r_t^{D,BoJ}D_{t-1}^{BoJ}$ is judgeable to be quite small, and the BoJ's returns on net assets are almost equivalent to its PFB.

$$PFB_t^{BoJ,net} = R_t^{BoJ,net} + r_t^{D,BoJ}D_{t-1}^{BoJ} - r_t^{D,BoJ}A_{t-1}^{BoJ} \approx R_t^{BoJ,net}$$

Figure 3 depicts the time-series of a net liability version of the PFB without unrealized returns for the GG ($PFB_t^{GG,net,R}$, a red solid line), its net liability version with them ($PFB_t^{GG,net}$, a black solid line), and its gross liability version ($PFB_t^{GG,gross}$, a red dotted line).

(insert Figure 3)

The series of $PFB_t^{GG,net,R}$ and $PFB_t^{GG,gross}$, either of which does not include any unrealized return from gross assets, are quite close to each other. Both series are chronically negative for the entire sample period. The only difference between the two is that the latter is lower by net asset purchase if $\Delta A_t^{GG,R} > 0$.

However, once unrealized returns from gross assets are included in the PFB, the series become quite volatile, and often positive. In particular, the series of $PFB_t^{GG,net}$ frequently record large positive numbers from the mid-2010s.

Adjusted by one-period lagged gross liabilities D_{t-1}^{GG} and converted into annual rates, the sample averages of $\frac{PFB_t^{GG,net,R}}{D_{t-1}^{GG}}$, $\frac{PFB_t^{GG,net}}{D_{t-1}^{GG}}$, and $\frac{PFB_t^{GG,gross}}{D_{t-1}^{GG}}$ are computed as −2.1% with standard deviation 0.8%, −1.4% with 2.9%, and −3.5% with 1.4%, respectively.

We have two remarks on these sample averages. First, the sample average of $\frac{PFB_t^{GG,net}}{D_{t-1}^{GG}}$ is still negative even with unrealized returns included. Second, the sample average of $\frac{PFB_t^{GG,net,R}}{D_{t-1}^{GG}}$ serves as the upper bound for the average of the relative PFB appearing in equations (8) and (17), or $\mathrm{E}\left(\frac{T^{GG}-G^{GG}}{D_{-1}^{GG}}\right)$. Thus, the theoretically consistent present value of the future PFB is largely negative.

As a byproduct of equation (1'), we can compute the series of quarterly interest rates on GG's liabilities (r_t^D) given D_{t-1}^{GG}. As shown in Figure 4, the borrowing rate for the GG declined substantially. The quarterly rate was around 0.7% in the late 1990s, but it decreased to 0.15% in the early 2020s.

(insert Figure 4)

3.3. Interpreting the observed PFB in terms of the β-CAPM regression

Let us interpret the observed PFB in terms of β-CAPM. Excess market returns, $r_t^m - r_t^D$, are constructed as follows. Market returns from Nikkei 225 are computed on a quarter-end to quarter-end basis. Quarterly dividend returns from the first section, or the prime section of the Tokyo Stock Exchange are added to market returns, while quarterly yields on one-year JGBs are subtracted from them. The sample average of $r_t^m - r_t^D$ is 8.1% at annual rates with standard deviation 20.4%. Thus, Sharpe ratio is 0.397.

Both $\frac{PFB_t^{GG,net,R}}{D_{t-1}^{GG}}$ and $\frac{PFB_t^{GG,net}}{D_{t-1}^{GG}}$ are regressed on $r_t^m - r_t^D$. $\beta^{GG,R}$ and $\beta^{GG,UR}$ are the estimated coefficient on $r_t^m - r_t^D$, while $\alpha^{GG,R}$ and $\alpha^{GG,UR}$ are the estimated constant term. Estimation results are reported in Table 1.

(insert Table 1)

Figures 5-1 and 5-2 draw a scatter diagram of excess market returns as X axis, and $\frac{PFB_t^{GG,net,R}}{D_{t-1}^{GG}}$ or $\frac{PFB_t^{GG,net}}{D_{t-1}^{GG}}$ as Y axis. As $\beta^{GG,R} < 0$ implies, $\frac{PFB_t^{GG,net,R}}{D_{t-1}^{GG}}$ is negatively, though insignificantly, correlated with $r_t^m - r_t^D$. Such slight negative correlation may be interpretable as less cyclical tax revenues T_t^{GG}. Significantly negative constant ($\alpha^{GG,R} < 0$), however, suggests that $T_t^{GG} - G_t^{GG}$ is largely negative on the average.

(insert Figure 5-1)

(insert Figure 5-2)

Once unrealized returns are added to the PFB, estimated β^{GG} changes from weakly negative to significantly positive, while estimated α^{GG} does not change that much, and remains negative. That is, the average PFB improves to the extent that the GG takes market risks. At the average excess market returns 2.0% as quarterly rates, the average relative PFB improves from −0.52% to −0.36%, and it is still negative. The GG seemed not to take market risk that much.

In terms of financial investment by the GG, given α^{GG} is close to $\alpha^{GG,R}$, the GG did not exploit effective arbitrage opportunities with positive α, while given $\beta^{GG} > 0 > \beta^{GG,R}$, the GG held assets with high β. Let us below explore how financial investment by the GG affects the β-CAPM regression.

As equations (3'-2-1) and (3'-2-2) show, unrealized returns of the GG are decomposed into those of the CLGs and those of the SSFs. With unrealized returns adjusted by one-period lagged gross assets ($\frac{PFB_t^{CLGs,UR}}{A_{t-1}^{CLGs}}$ and $\frac{PFB_t^{SSfs,UR}}{A_{t-1}^{SSFs}}$), the average of the two unrealized returns is 1.0% at annual rates with standard deviation 7.7% for the CLGs, and 0.6% with 4.9% for the SSFs. Thus, Sharpe ratios for the CLGs and SSFs are 0.131 and 0.127, both of which are quite low compared with that of excess market returns $r_t^m - r_t^D$, 0.397.

Those unrealized returns are scattered against $r_t^m - r_t^D$ in Figures 6-1 and 6-2. According to Table 1, estimated α is almost zero for the CLGs and the SSFs, meaning that the Euler equation still holds with respect to excess returns for both sectors, and there is no room for both to exploit arbitrage opportunities. On the other hand, estimated β is 0.1374 for the CLGs, and 0.1831 for the SSFs. The latter estimation results imply that even the SSFs, which have been regarded as active institutional investors, did not actually hold high β assets.

(insert Figure 6-1)

(insert Figure 6-2)

Let us look at realized and unrealized returns on net assets held by the BoJ, $\frac{R_t^{BoJ,net}}{A_{t-1}^{BoJ}-D_{t-1}^{BoJ}}$. Those returns are scattered against $r_t^m - r_t^D$ in Figure 6-3. As shown in Table 1, the estimation result of the BoJ is quite different from that of the CLGs or the SSFs. Estimated β of the BoJ is 0.1003; it is smaller than those of the CLGs (0.1374) and SSFs (0.1831). But estimated α is 0.0291; it is much higher than almost zero α of the CLGs and SSFs. According to these estimation results, the BoJ exploited effective arbitrage opportunities with positive α, and it earned high excess returns without taking much market risk.

Such arbitrage opportunities might have arisen from almost zero financing costs on the one hand, and extremely active investment in the long-term JGBs on the other hand, the latter of which generated huge capital gains as ten-year or longer yields declined substantially in the 2010s. As shown below, however, high excess returns from BoJ's arbitrage behavior have little impact on the PFB of the IG.

(insert Figure 6-3)

3.4. Computing α and β of IG's PFB

We below compute α and β for the IG, consisting of the GG and the BoJ, under some simplifying assumptions. The PFB relative to gross liabilities for the IG ($\frac{PFB_t^{IG,net}}{D_{t-1}^{GG}}$) is decomposed as follows.

$$\frac{PFB_t^{IG,net}}{D_{t-1}^{GG}} = \frac{PFB_t^{GG,net,R}}{D_{t-1}^{GG}} + \frac{PFB_t^{CLGs,UR}}{A_{t-1}^{CLGs}}\frac{A_{t-1}^{CLGs}}{D_{t-1}^{GG}} + \frac{PFB_t^{CLGs,UR}}{A_{t-1}^{SSFs}}\frac{A_{t-1}^{SSFs}}{D_{t-1}^{GG}} + \frac{R_t^{BoJ,net}}{A_{t-1}^{BoJ}-D_{t-1}^{BoJ}}\frac{A_{t-1}^{BoJ}-D_{t-1}^{BoJ}}{D_{t-1}^{GG}}$$

The above equation is heroically approximated as

$$\frac{PFB_t^{IG,net}}{D_{t-1}^{GG}} \approx \frac{PFB_t^{GG,net,R}}{D_{t-1}^{GG}} + \frac{PFB_t^{CLGs,UR}}{A_{t-1}^{CLGs}}\mathrm{E}\left(\frac{A_{t-1}^{CLGs}}{D_{t-1}^{GG}}\right) + \frac{PFB_t^{CLGs,UR}}{A_{t-1}^{SSFs}}\mathrm{E}\left(\frac{A_{t-1}^{SSFs}}{D_{t-1}^{GG}}\right) + \frac{R_t^{BoJ,net}}{A_{t-1}^{BoJ}-D_{t-1}^{BoJ}}\mathrm{E}\left(\frac{A_{t-1}^{BoJ}-D_{t-1}^{BoJ}}{D_{t-1}^{GG}}\right). \quad (25)$$

If excess returns ($r_t^{CLGs,UR} - r_t^D$, $r_t^{SSFs,UR} - r_t^D$, and $\frac{R_t^{BoJ,net}}{A_{t-1}^{BoJ}-D_{t-1}^{BoJ}}$) are uncorrelated with the asset share ($\frac{A_{t-1}^{CLGs}}{D_{t-1}^{GG}}$, $\frac{A_{t-1}^{SSFs}}{D_{t-1}^{GG}}$, and $\frac{A_{t-1}^{BoJ}-D_{t-1}^{BoJ}}{D_{t-1}^{GG}}$) as assumed in Sections 2-1 and 2-2, equation (25) is an exact representation. As shown in Figure 7, however, $\frac{A_{t-1}^{CLGs}}{D_{t-1}^{GG}}$, $\frac{A_{t-1}^{SSFs}}{D_{t-1}^{GG}}$, and $\frac{A_{t-1}^{BoJ}-D_{t-1}^{BoJ}}{D_{t-1}^{GG}}$ increased as asset prices appreciated from the mid-2010s, and equation (19) is quite bold approximation. Nevertheless, we below assign the sample average 0.300 to $\mathrm{E}\left(\frac{A_{t-1}^{CLGs}}{D_{t-1}^{GG}}\right)$, 0.252 to $\mathrm{E}\left(\frac{A_{t-1}^{SSFs}}{D_{t-1}^{GG}}\right)$, and 0.015 to $\mathrm{E}\left(\frac{A_{t-1}^{BoJ}-D_{t-1}^{BoJ}}{D_{t-1}^{GG}}\right)$, respectively.

(insert Figure 7)

We have some comments on Figure 7. First, neither $\frac{A_{t-1}^{CLGs}}{D_{t-1}^{GG}}$ nor $\frac{A_{t-1}^{SSFs}}{D_{t-1}^{GG}}$ displays any monotonic upward trend for the entire sample period. $\frac{A_{t-1}^{CLGs}}{D_{t-1}^{GG}}$ moves between 0.25 and 0.35, while $\frac{A_{t-1}^{SSFs}}{D_{t-1}^{GG}}$ has a downward trend accompanied by an upward trend. Second, BoJ's net position $\frac{A_{t-1}^{BoJ}-D_{t-1}^{BoJ}}{D_{t-1}^{GG}}$ is rather small relative to GG's gross liabilities.

Given approximation (25), α^{IG} and β^{IG} are expressed as follows.

$$\alpha^{IG} = \alpha^{GG,R} + \alpha^{CLG,UR}\mathrm{E}\left(\frac{A_{t-1}^{CLG}}{D_{t-1}^{GG}}\right) + \alpha^{SSF,UR}\mathrm{E}\left(\frac{A_{t-1}^{SSF}}{D_{t-1}^{GG}}\right) + \alpha^{BoJ,net}\mathrm{E}\left(\frac{A_{t-1}^{BoJ}-D_{t-1}^{BoJ}}{D_{t-1}^{GG}}\right) \quad (26)$$

$$\beta^{IG} = \beta^{GG,R} + \beta^{CLG,UR}\mathrm{E}\left(\frac{A_{t-1}^{CLG}}{D_{t-1}^{GG}}\right) + \beta^{SSF,UR}\mathrm{E}\left(\frac{A_{t-1}^{SSF}}{D_{t-1}^{GG}}\right) + \beta^{BoJ,net}\mathrm{E}\left(\frac{A_{t-1}^{BoJ}-D_{t-1}^{BoJ}}{D_{t-1}^{GG}}\right) \quad (27)$$

According to equation (22), only αs in the right-hand side of equation (26) are significant in the evaluation of the discounted PFB, while as equation (23) implies, not only αs in that of equation (26), but also βs in that of equation (27) matter. Given that estimated β^{T-G} and αs ($\alpha^{CLG,UR}$, $\alpha^{SSF,UR}$, and $\alpha^{BoJ,net}\mathrm{E}\left(\frac{A_{t-1}^{BoJ}-D_{t-1}^{BoJ}}{D_{t-1}^{GG}}\right)$) are rather small as reported in Table 1, $\mathrm{E}\left(\frac{T^{GG}-G^{GG}}{D_{-1}^{GG}}\right) - \beta^{T-G}E(r^m - r^D) + \alpha^{GG}\frac{A_{-1}^{GG}}{D_{-1}^{GG}} \approx \mathrm{E}\left(\frac{T^{GG}-G^{GG}}{D_{-1}^{GG}}\right)$, and parameters of the β-CAPM regression have almost no impact on the discounted PFB in equation (22). Thus, we below follow the interpretation of equation (23), although the combination of β-CAPM and constant discounting is quite ad-hoc.

As demonstrated by Table 2, substituting estimated αs and βs from Table 1, and the sample averages into the right-hand side of equations (26) and (27), α^{IG} is computed as -0.0053, and β^{IG}

as 0.0837. α^{IG} is close to $\alpha^{GG,net}$ (-0.0051), while β^{IG} is a little higher than $\beta^{GG,net}$ (0.0770).

(insert Table 2)

Figure 8 draws four linear functions with

A) a pair of $\alpha^{GG,R}$ and $\beta^{GG,R}$,

B) a pair of $\alpha^{GG,R} + 0.3\alpha^{CLG,UR}$ and $\beta^{GG,R} + 0.3\beta^{CLG,UR}$,

C) a pair of $\alpha^{GG,R} + 0.3\alpha^{CLG,UR} + 0.252\alpha^{SSF,UR}$ and $\beta^{GG,R} + 0.3\beta^{CLG,UR} + 0.252\beta^{SSF,UR}$, and

D) a pair of $\alpha^{GG,R} + 0.3\alpha^{CLG,UR} + 0.252\alpha^{SSF,UR} + 0.015\alpha^{BoJ,net}$ and $\beta^{GG,R} + 0.3\beta^{CLG,UR} + 0.252\beta^{SSF,UR} + 0.015\beta^{BoJ,net}$.

(insert Figure 8)

Figure 8 illuminates how risky investment by the CLGs, the SSFs, and the BoJ affects the risk-return structure on IG's gross liabilities. Without any risky investment, a linear line is slightly downward-sloping. Including risky investment one by one, however, the slope of linear lines is steeper and steeper from -0.0051 to 0.0361 by CLGs' investment, to 0.0822 by SSFs', and to 0.0837 by BoJ's. The constant terms remain around 0.005.

If four lines are evaluated at the expected excess market return 2% as quarter rates, the expected excess return on GI's gross liabilities improves from -0.0052 to -0.0045, -0.0041, and -0.0036, respectively. Each increment ranges between 0.0004 and 0.0007, and it is rather small. Even with all risky investments included, the expected excess return remains negative. It means that the government never yields positive cash flow on the average although parameters of the β-CAPM regression for excess returns on risky assets held by the CLGs and the SSFs, and net assets held by the BoJ are considered to the fullest.

In conclusion, the present value of the future $PFB_t^{IG,net}$ is far short of the current valuation of the net liabilities as follows.

$$1 - \mathrm{E}\left(\frac{A_{-1}^{GG}}{D_{-1}^{GG}}\right) > 0 > \frac{1}{r^D}\left[\alpha^{IG} + \beta^{IG}\mathrm{E}(r^m - r^D)\right]$$

In this way, holding high β assets and claims does not help sustain the huge net liabilities.

3.5. Should the Japanese government take much more market risk to make the expected PFB

positive?

Let us compare the above estimation results with Chien et al.'s calibration results. As discussed in Section 2.3, Chien et al. set all αs at zero. In our estimation, both $\alpha^{CLG,UR}$ and $\alpha^{SSF,UR}$ are close to zero as well. But $\alpha^{GG,R}$ is significantly negative, while $\alpha^{BoJ,net}$ is positive though insignificant. On the other hand, they calibrate β^{A} (β of gross assets) to be 0.5. Such β is much higher than our estimates $\beta^{CLG,UR} = 0.14$, $\beta^{SSF,UR} = 0.18$, or $\beta^{BoJ,net} = 0.10$. In addition, their $(1 - 0.66) \times \beta^{S} = 0.15$ (β of the surplus claim) suggests that the primary surplus is highly cyclical, while our $\beta^{T-G} = -0.005$ implies that it is slightly counter-cyclical. Given their calibration, the expected PFB is well above zero (2.4% per year), and as far as the expected value is concerned, the current net liabilities are sustainable with the future PFB. But their calibration is not compatible with the standard β-CAPM regression based on the quarterly observations of the sample period between 1998 and 2024.

According to the exercises in Sections 3.3 and 3.4, the IG can make the expected PFB positive only if it takes much more market risk. Suppose that $\mathrm{E}\left(\frac{T^{GG}-G^{GG}}{D_{-1}^{GG}}\right) = -0.005$, $\mathrm{E}\left(\frac{A_{t-1}^{CLG}}{D_{t-1}^{GG}} + \frac{A_{t-1}^{SSF}}{D_{t-1}^{GG}} + \frac{A_{t-1}^{BoJ}-D_{t-1}^{BoJ}}{D_{t-1}^{GG}}\right) = 0.6$, and $\mathrm{E}(r_t^m - r_t^D) = 0.02$. If the IG bodies hold highly risky portfolios with $\beta \geq 0.42$, which is five times as large as the current $\beta \approx 0.08$, and comparable to Chien et al.'s calibration of $\beta^{A} = 0.5$, then the expected PFB turns out to be positive. In Figure 8, a black dotted line represents such high β investment by the IG.

Do the taxpayers desire such a dramatic change in the government's asset portfolios? Note that β^{GG} appears in equation (23) because not stochastic discounting, but constant discounting is applied. That is, the IG is risk neutral in the sense that they do not care for how volatile the government's portfolios are. Given quite conservative asset portfolios in the Japanese households, it is hard to imagine that the assumption employed in equation (23) is consistent with taxpayers' risk averse attitude.

Of course, there are alternative interpretations about hypothetical asymmetry between the government's risky portfolios and the households' conservative portfolios. First, the government and the taxpayers are corporative with each other. The households prefer risky portfolios to conservative ones, but they cannot hold them due to several constraints. Thus, the government would hold risky assets on behalf of the taxpayers.

Second, the government and the taxpayers are hostile to each other. Again, the households are forced to hold conservative portfolios due to several constraints. Many interest groups, including financial institutions, govern the government independently of the taxpayers' interest.

The government would hold risky assets at the sacrifice of the forced households, and on behalf of these interest groups. According to interpretations by Chien et al., younger and less financially sophisticated households correspond to those who are forced to hold conservative portfolios, while older and financially sophisticated households to those who belong to interest groups.

4. Conclusion

One important message from our theoretical and empirical investigation is summarized as follows. In the past decade, enormous amounts of unrealized capital gains were indeed reflected in the market valuation of risky assets and contributed to a substantial reduction in the net liabilities. It is this valuation improvement that Chien et al. pay a serious attention to. But the discounted PFB or its present value is directly determined not by the past realization of capital gains, but by the future possibility of capital gains as well as losses. Under stochastic discounting, positive (negative) excess returns are discounted more (less) heavily, and accordingly the discounted excess returns degenerate to zero. Then, βs of the surplus claim and the risky assets have no impact on the discounted PFB.

Under constant discounting in which both positive and negative excess returns are discounted equally, on the other hand, there is room for βs to affect not only the expected PFB, but also the discounted PFB. However, estimated βs of the risky assets held by the integrated government have limited impacts on the discounted PFB because the government is currently taking only the moderate level of β for risky investment. As demonstrated in Section 2, the current valuation of the net liabilities has to be sustained largely by the sequence of the conventional primary surplus $T_t^{GG} - G_t^{GG}$. Nevertheless, if the government pursues unreasonably high β for the dramatic improvement of the expected PFB as implicitly proposed by Chien et al., then the taxpayers are forced to take unbearably high risks without any improvement of the discounted PFB.

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Table 1: Estimation results of β-CAPM regression

	PFB without unrealized returns on gross liabilities for GG	PFB with realized returns on gross liabilities for GG	Unrealized excess returns on gross assets for CLGs	Unrealized excess returns on gross assets for SSFs	Realized and unrealized excess returns on net assets for BoJ
α	-0.0051 (0.0004)	-0.0051 (0.0012)	-0.0002 (0.0036)	-0.0021 (0.0016)	0.0291 (0.0186)
β	-0.0051 (0.0039)	0.0770 (0.0117)	0.1374 (0.0346)	0.1831 (0.0152)	0.1003 (0.1802)

Table 2: Impacts of β of risky investment of CLGs, SSFs, and BoJ on β of the integrated government

	Realized PFB on gross liabilities for GG	Unrealized excess returns on gross assets for CLGs	Unrealized excess returns on gross assets for SSFs	Realized and unrealized excess returns on net assets for BoJ	Computed β of the integrated government
(i) average of gross assets share/total gross liabilities	1.000	0.300	0.252	0.015	
(ii) estimated α	-0.0051	-0.0002	-0.0021	0.0291	sum of (iii)
(iii) estimated impact ((i) × (ii))	**-0.0051**	**-0.0001**	**-0.0005**	**0.0004**	**-0.0053**
(iv) estimated β	-0.0051	0.1374	0.1831	0.1003	sum of (V)
(v) estimated impact ((i) × (iv))	**-0.0051**	**0.0412**	**0.0461**	**0.0015**	**0.0837**

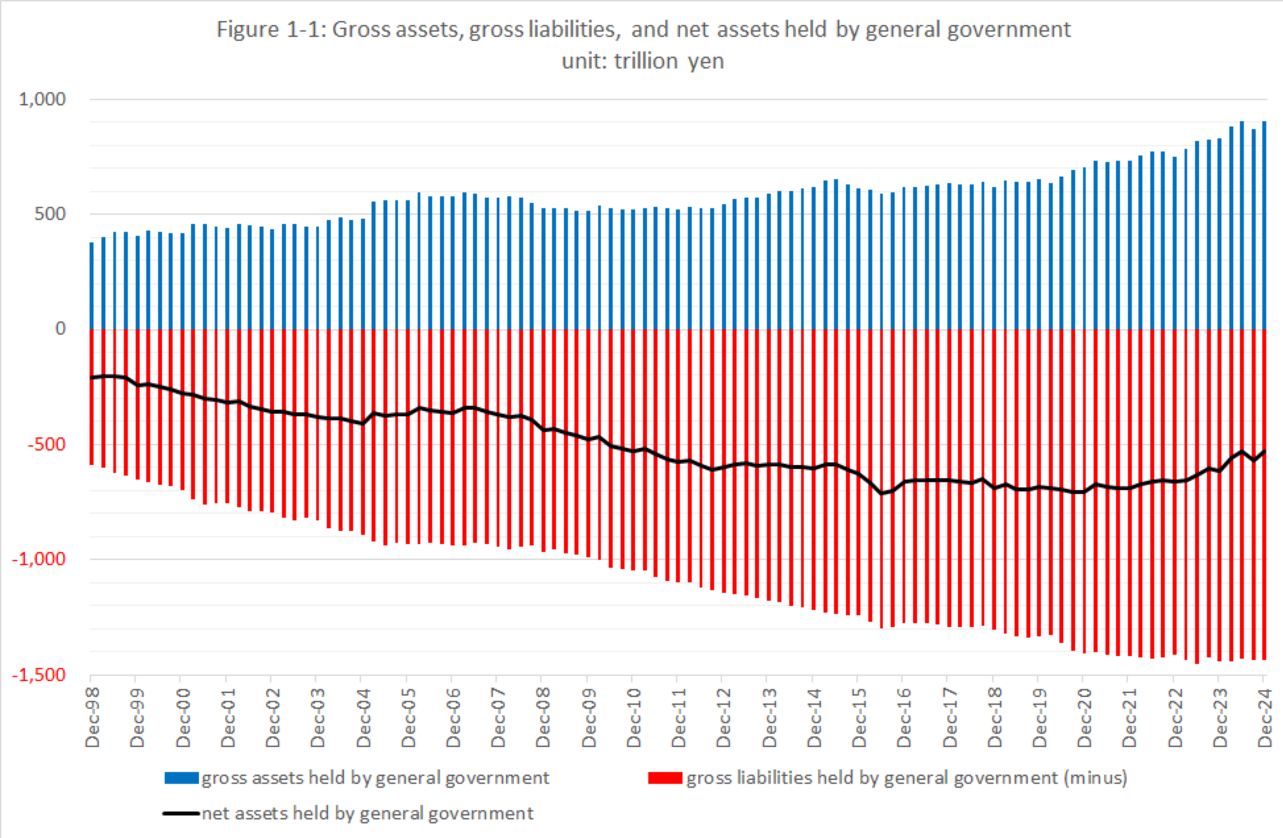


Figure 1-1: Gross assets, gross liabilities, and net assets held by general government
unit: trillion yen
1,000
500
0
-500
-1,000
-1,500
Dec-98
Dec-99
Dec-00
Dec-01
Dec-02
Dec-03
Dec-04
Dec-05
Dec-06
Dec-07
Dec-08
Dec-09
Dec-10
Dec-11
Dec-12
Dec-13
Dec-14
Dec-15
Dec-16
Dec-17
Dec-18
Dec-19
Dec-20
Dec-21
Dec-22
Dec-23
Dec-24
gross assets held by general government
gross liabilities held by general government (minus)
net assets held by general government

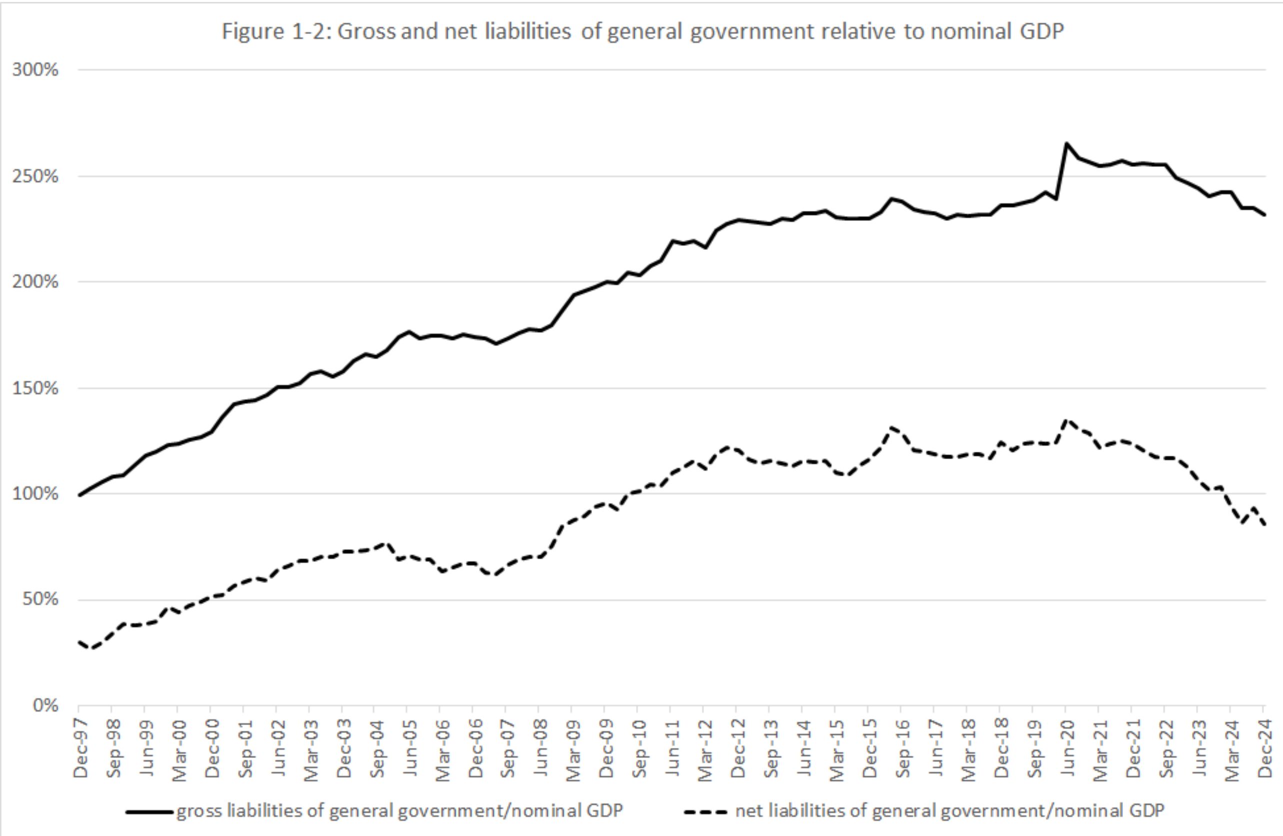


Figure 1-2: Gross and net liabilities of general government relative to nominal GDP
300%
250%
200%
150%
100%
50%
0%
Dec-97
Sep-98
Jun-99
Mar-00
Dec-00
Sep-01
Jun-02
Mar-03
Dec-03
Sep-04
Jun-05
Mar-06
Dec-06
Sep-07
Jun-08
Mar-09
Dec-09
Sep-10
Jun-11
Mar-12
Dec-12
Sep-13
Jun-14
Mar-15
Dec-15
Sep-16
Jun-17
Mar-18
Dec-18
Sep-19
Jun-20
Mar-21
Dec-21
Sep-22
Jun-23
Mar-24
Dec-24
gross liabilities of general government/nominal GDP
net liabilities of general government/nominal GDP

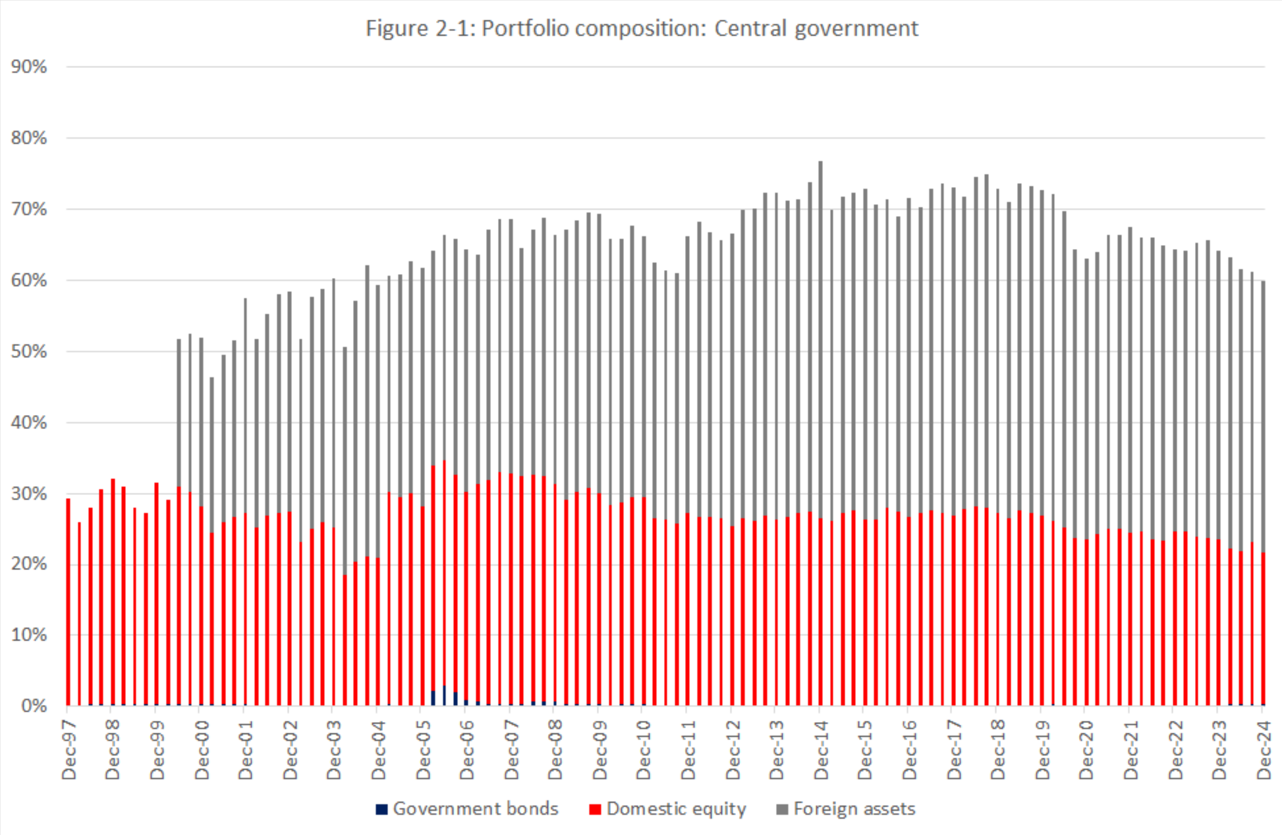


Figure 2-1: Portfolio composition: Central government
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
Dec-97 Dec-98 Dec-99 Dec-00 Dec-01 Dec-02 Dec-03 Dec-04 Dec-05 Dec-06 Dec-07 Dec-08 Dec-09 Dec-10 Dec-11 Dec-12 Dec-13 Dec-14 Dec-15 Dec-16 Dec-17 Dec-18 Dec-19 Dec-20 Dec-21 Dec-22 Dec-23 Dec-24
Government bonds
Domestic equity
Foreign assets

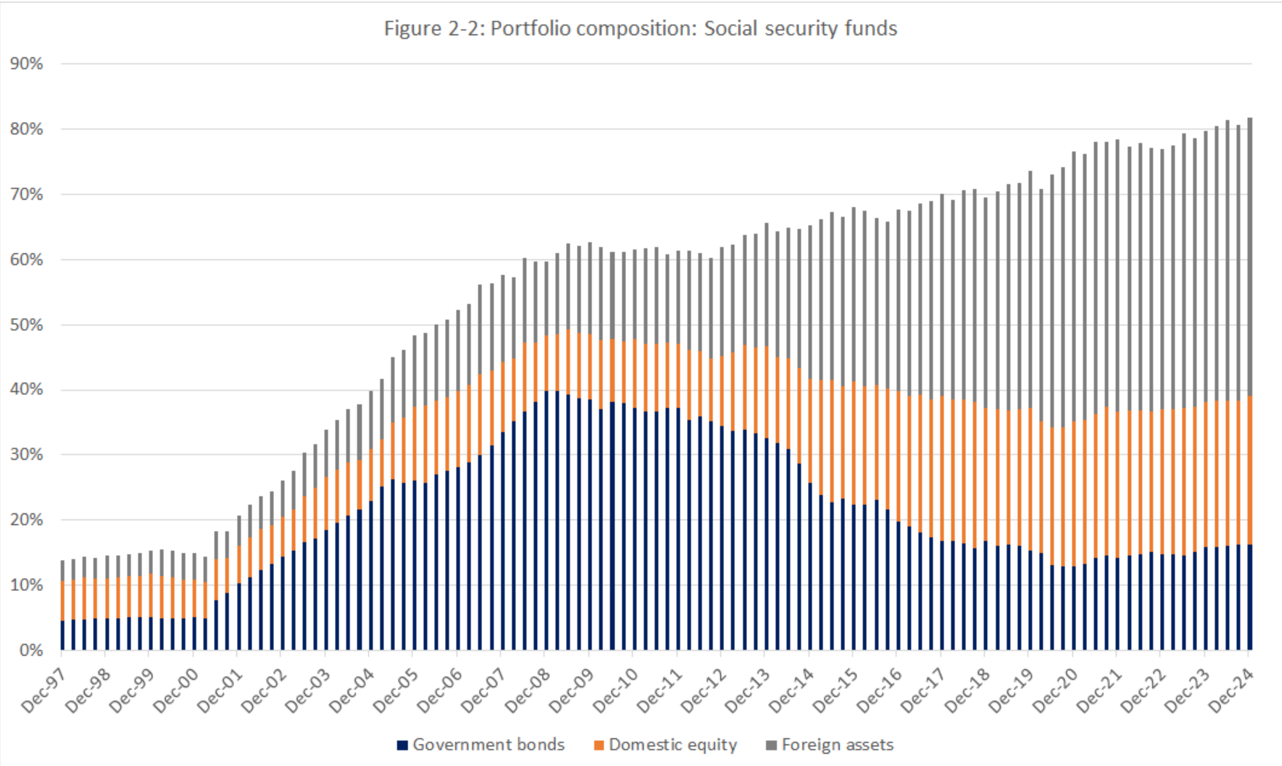


Figure 2-2: Portfolio composition: Social security funds
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
Dec-97 Dec-98 Dec-99 Dec-00 Dec-01 Dec-02 Dec-03 Dec-04 Dec-05 Dec-06 Dec-07 Dec-08 Dec-09 Dec-10 Dec-11 Dec-12 Dec-13 Dec-14 Dec-15 Dec-16 Dec-17 Dec-18 Dec-19 Dec-20 Dec-21 Dec-22 Dec-23 Dec-24
Government bonds
Domestic equity
Foreign assets

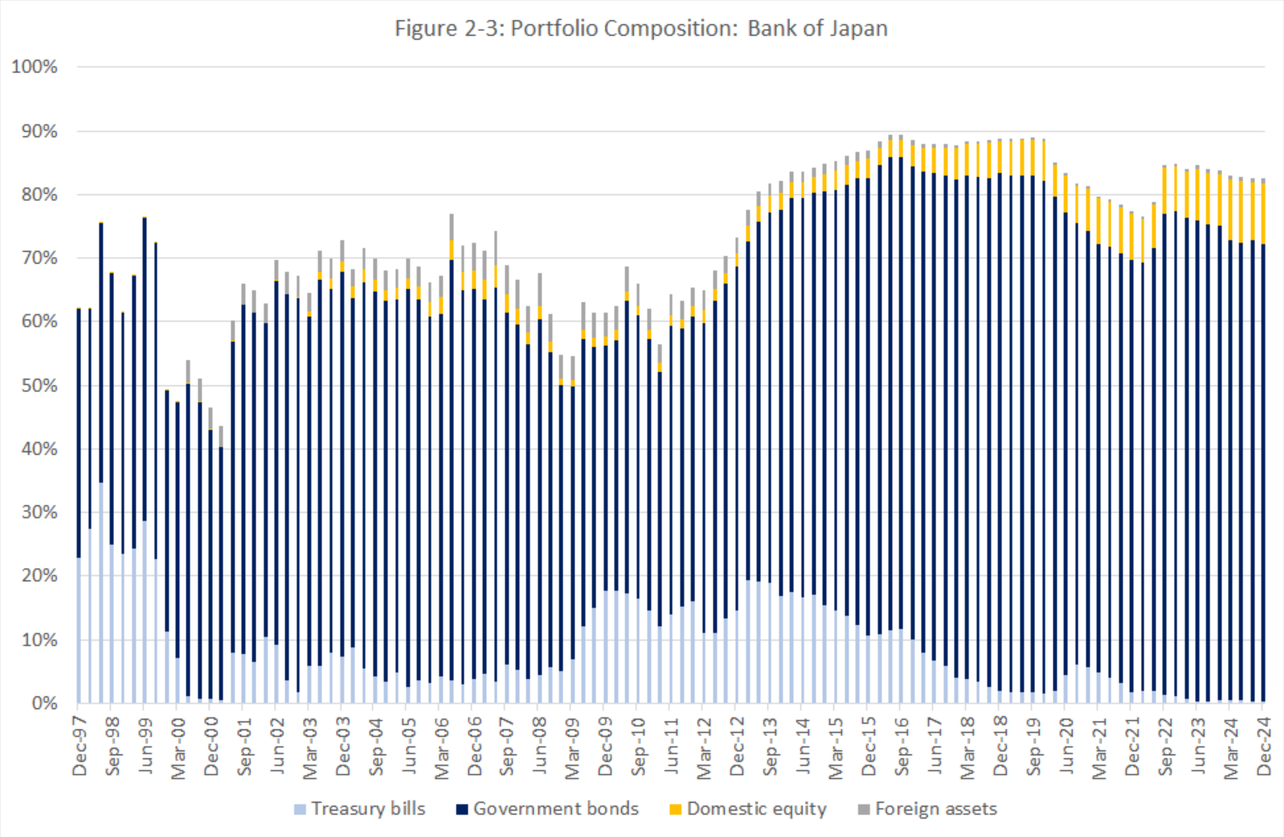


Figure 2-3: Portfolio Composition: Bank of Japan
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
Dec-97
Sep-98
Jun-99
Mar-00
Dec-00
Sep-01
Jun-02
Mar-03
Dec-03
Sep-04
Jun-05
Mar-06
Dec-06
Sep-07
Jun-08
Mar-09
Dec-09
Sep-10
Jun-11
Mar-12
Dec-12
Sep-13
Jun-14
Mar-15
Dec-15
Sep-16
Jun-17
Mar-18
Dec-18
Sep-19
Jun-20
Mar-21
Dec-21
Sep-22
Jun-23
Mar-24
Dec-24
Treasury bills
Government bonds
Domestic equity
Foreign assets

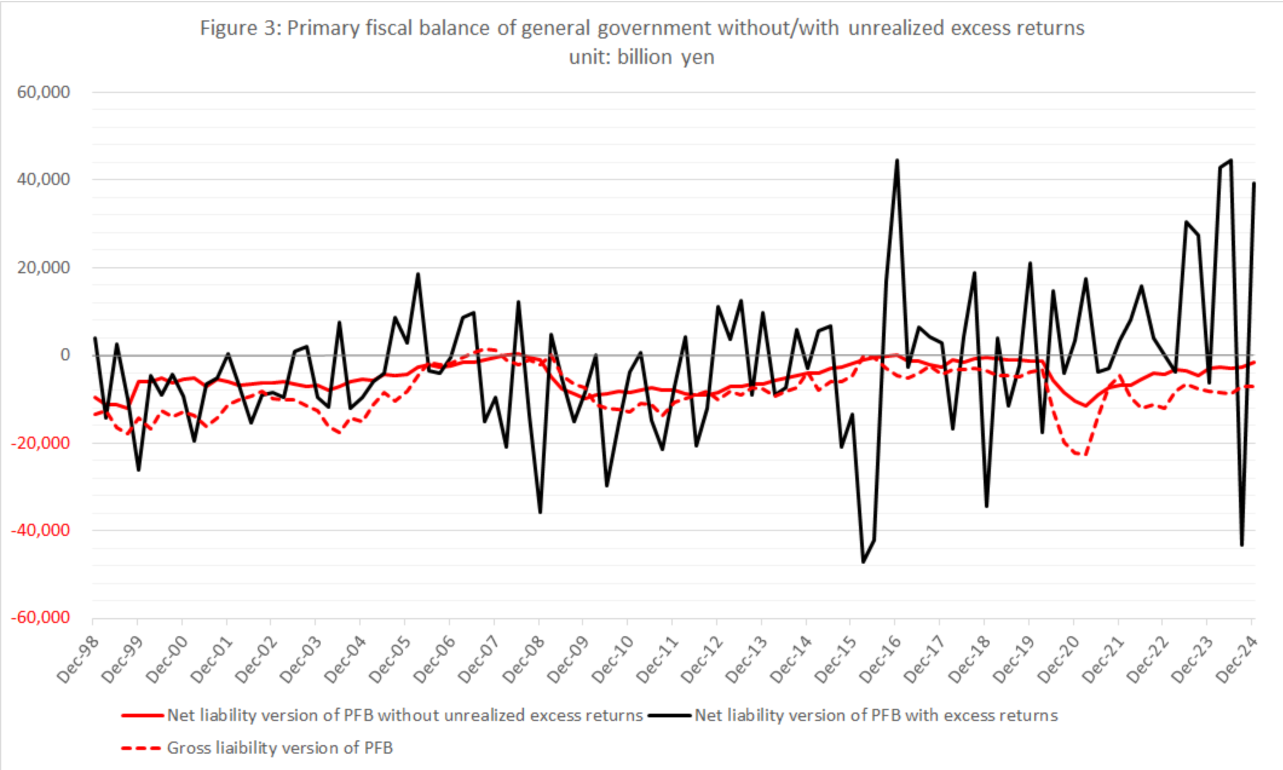


Figure 3: Primary fiscal balance of general government without/with unrealized excess returns
unit: billion yen
60,000
40,000
20,000
0
-20,000
-40,000
-60,000
Dec-98
Dec-99
Dec-00
Dec-01
Dec-02
Dec-03
Dec-04
Dec-05
Dec-06
Dec-07
Dec-08
Dec-09
Dec-10
Dec-11
Dec-12
Dec-13
Dec-14
Dec-15
Dec-16
Dec-17
Dec-18
Dec-19
Dec-20
Dec-21
Dec-22
Dec-23
Dec-24
Net liability version of PFB without unrealized excess returns
Net liability version of PFB with excess returns
Gross liability version of PFB

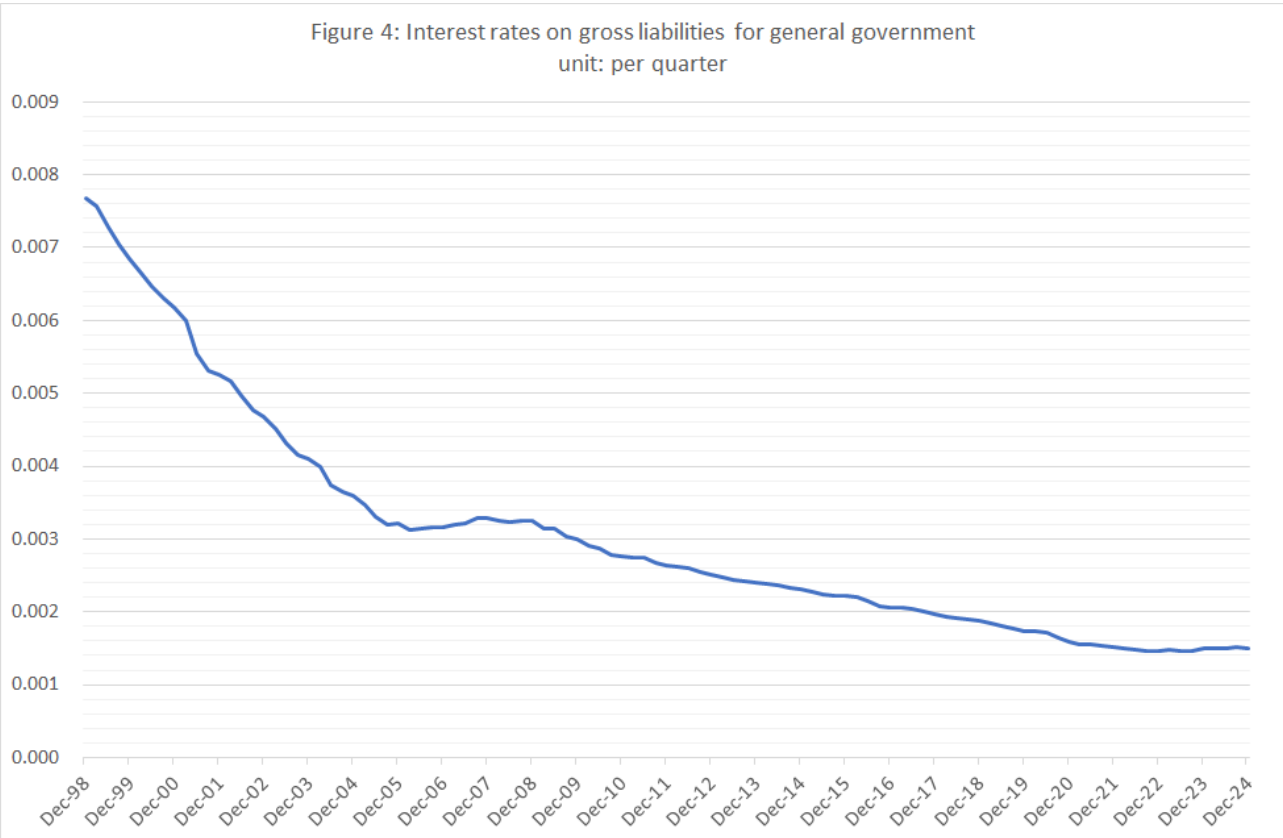


Figure 4: Interest rates on gross liabilities for general government
unit: per quarter
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0.000
Dec-98
Dec-99
Dec-00
Dec-01
Dec-02
Dec-03
Dec-04
Dec-05
Dec-06
Dec-07
Dec-08
Dec-09
Dec-10
Dec-11
Dec-12
Dec-13
Dec-14
Dec-15
Dec-16
Dec-17
Dec-18
Dec-19
Dec-20
Dec-21
Dec-22
Dec-23
Dec-24

Figure 5-1: Realized primary fiscal balance on general government's gross liabilities against excess equity returns

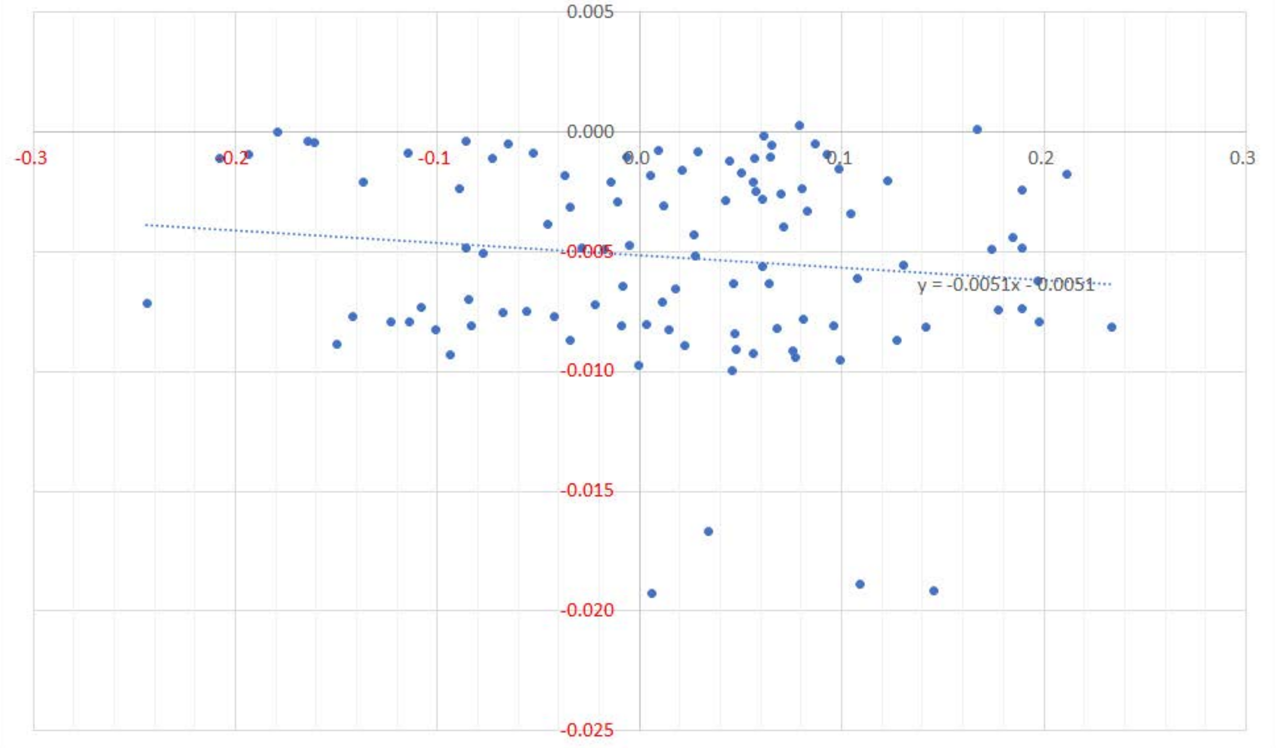
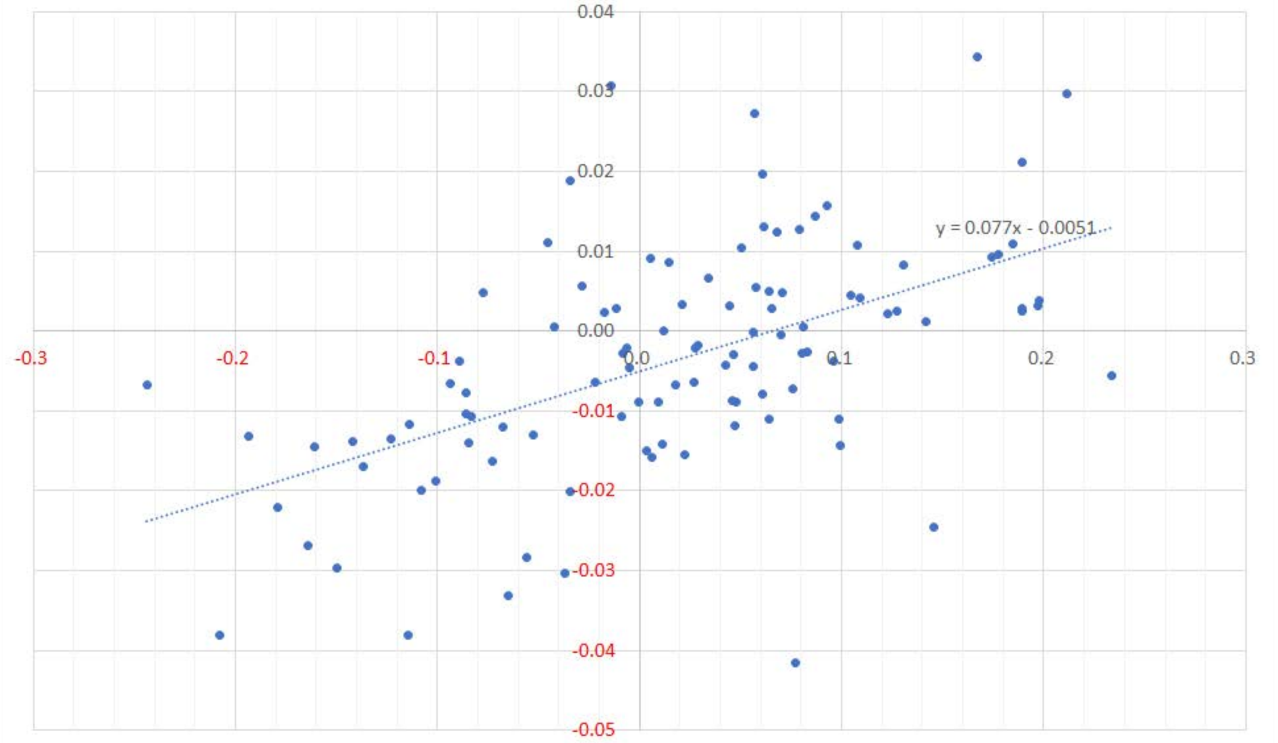


Figure 5-2: Realized and unrealized primary fiscal balance on general government's gross liabilities against excess equity returns



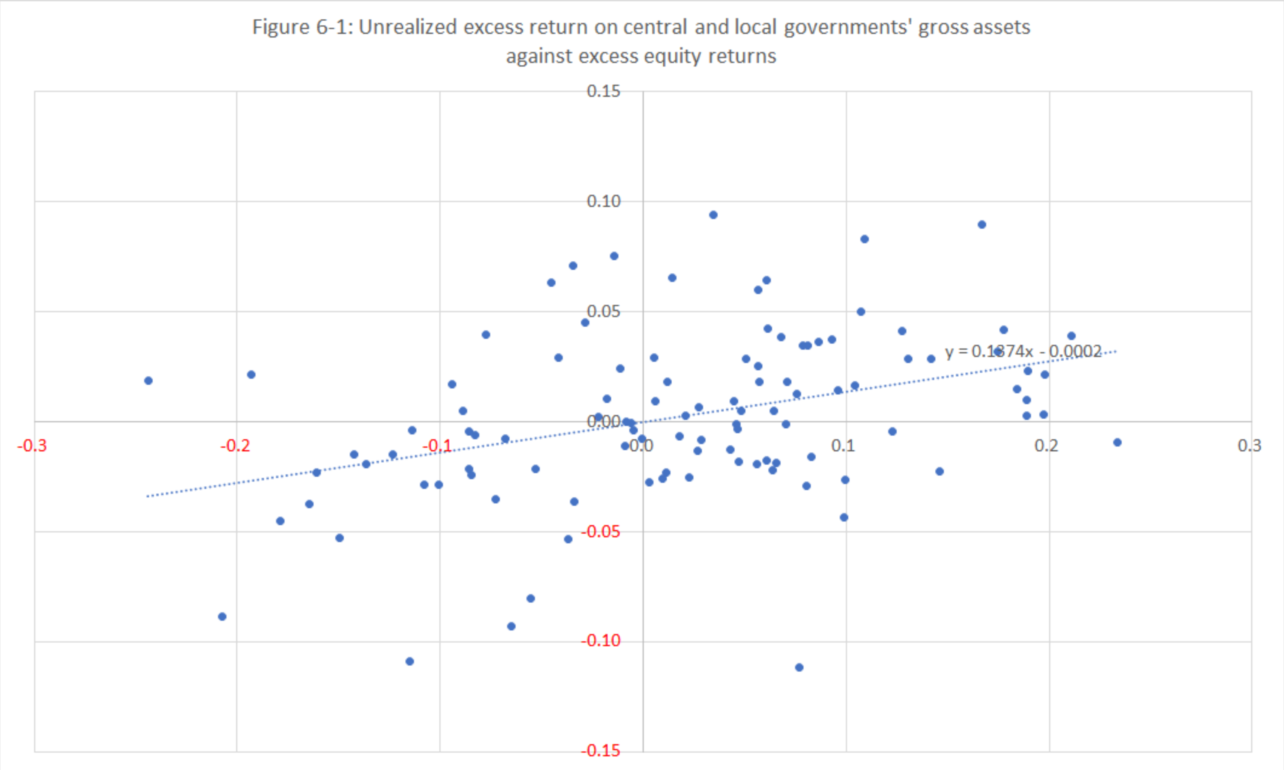


Figure 6-1: Unrealized excess return on central and local governments' gross assets against excess equity returns
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
-0.3
-0.2
-0.1
0.0
0.1
0.2
0.3
y = 0.1874x - 0.0002

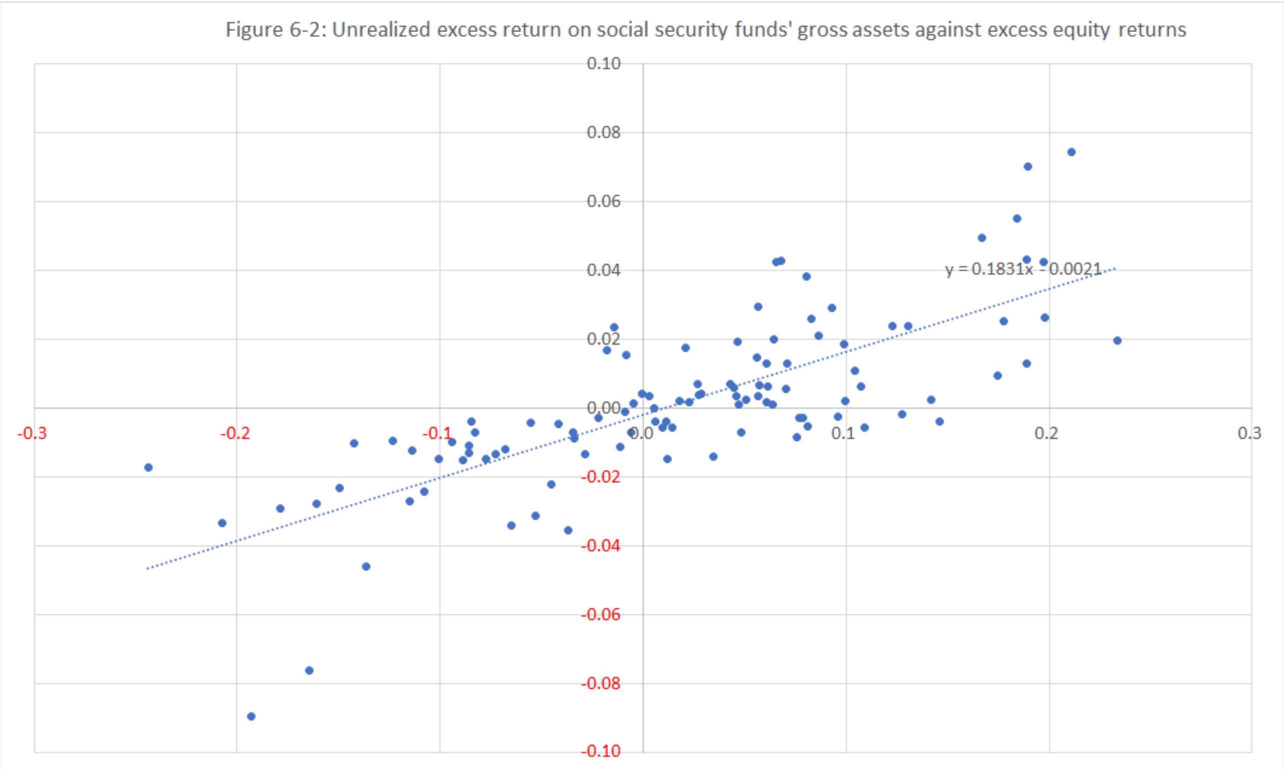


Figure 6-2: Unrealized excess return on social security funds' gross assets against excess equity returns
0.10
0.08
0.06
0.04
0.02
0.00
-0.02
-0.04
-0.06
-0.08
-0.10
-0.3
-0.2
-0.1
0.0
0.1
0.2
0.3
y = 0.1831x - 0.0021

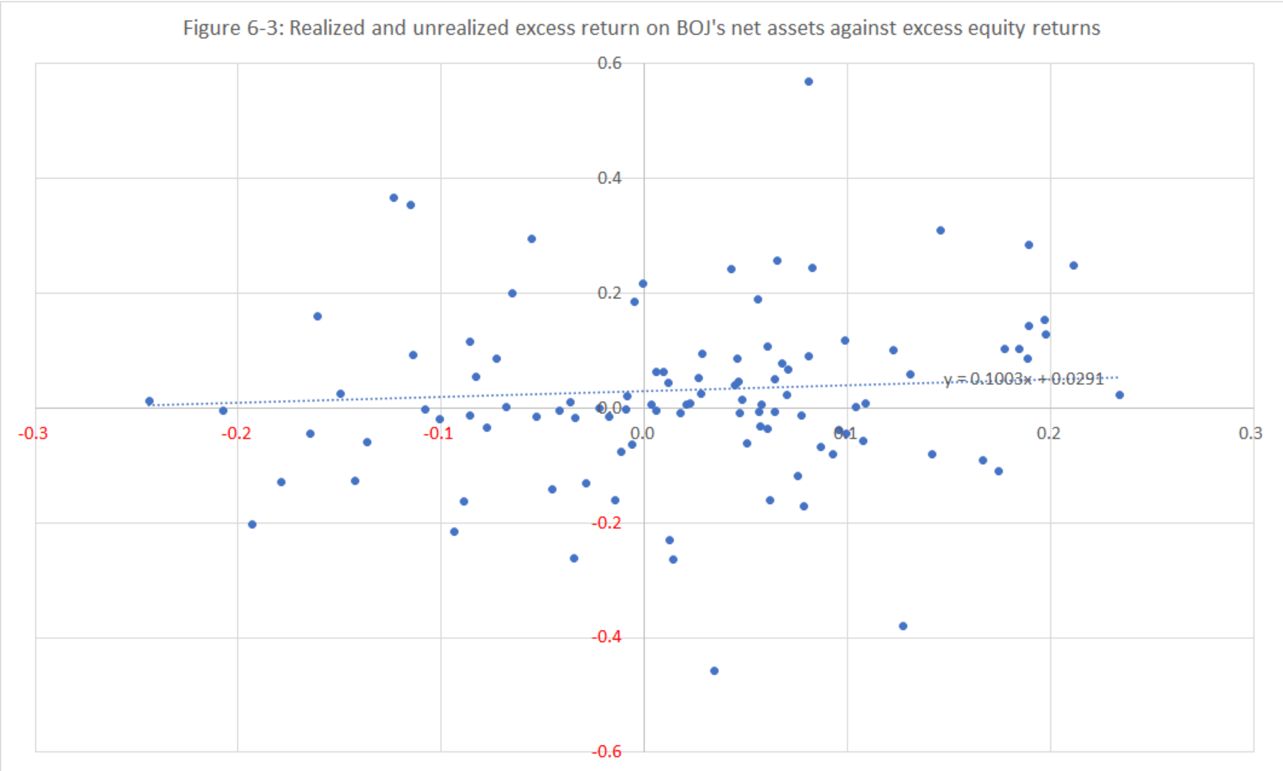


Figure 6-3: Realized and unrealized excess return on BOJ's net assets against excess equity returns
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.3
-0.2
-0.1
0.0
0.1
0.2
0.3
y = 0.1003x + 0.0291

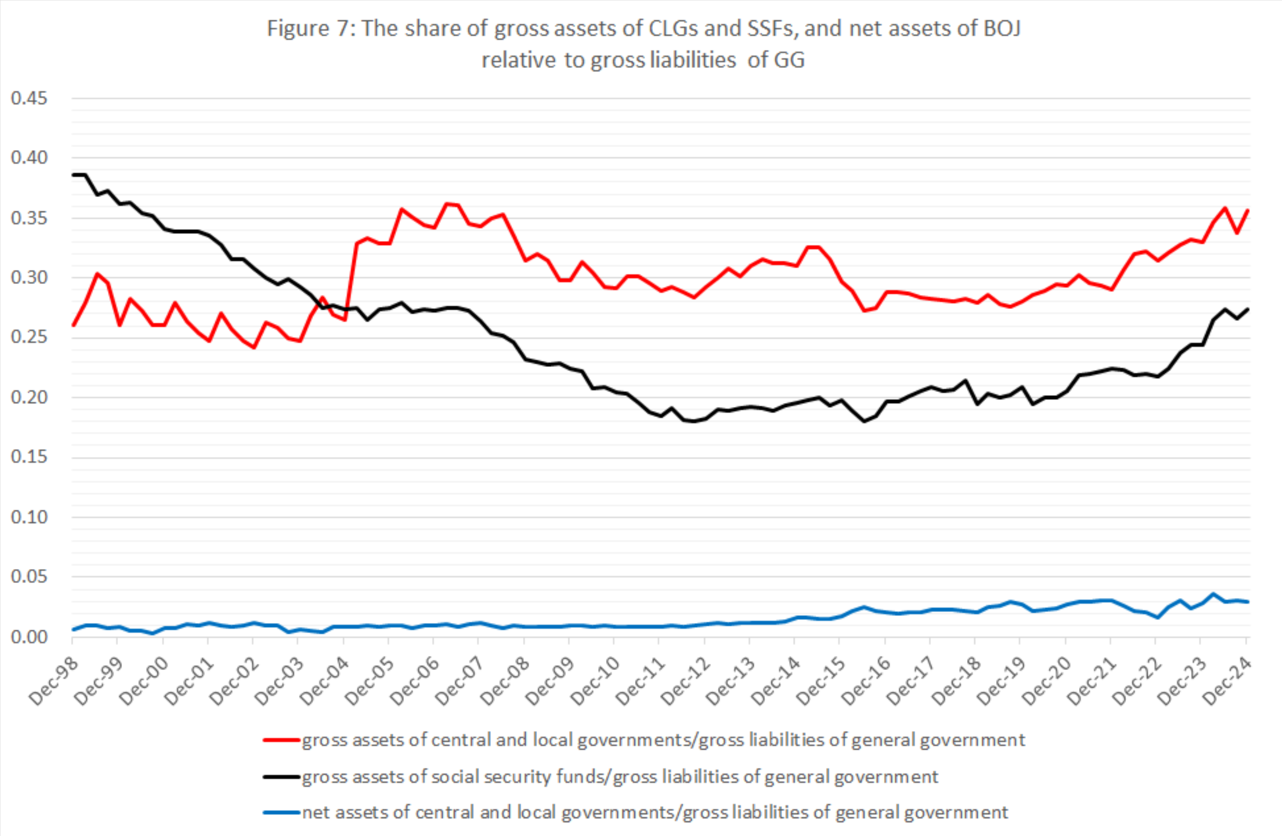


Figure 7: The share of gross assets of CLGs and SSFs, and net assets of BOJ relative to gross liabilities of GG
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
Dec-98
Dec-99
Dec-00
Dec-01
Dec-02
Dec-03
Dec-04
Dec-05
Dec-06
Dec-07
Dec-08
Dec-09
Dec-10
Dec-11
Dec-12
Dec-13
Dec-14
Dec-15
Dec-16
Dec-17
Dec-18
Dec-19
Dec-20
Dec-21
Dec-22
Dec-23
Dec-24
gross assets of central and local governments/gross liabilities of general government
gross assets of social security funds/gross liabilities of general government
net assets of central and local governments/gross liabilities of general government

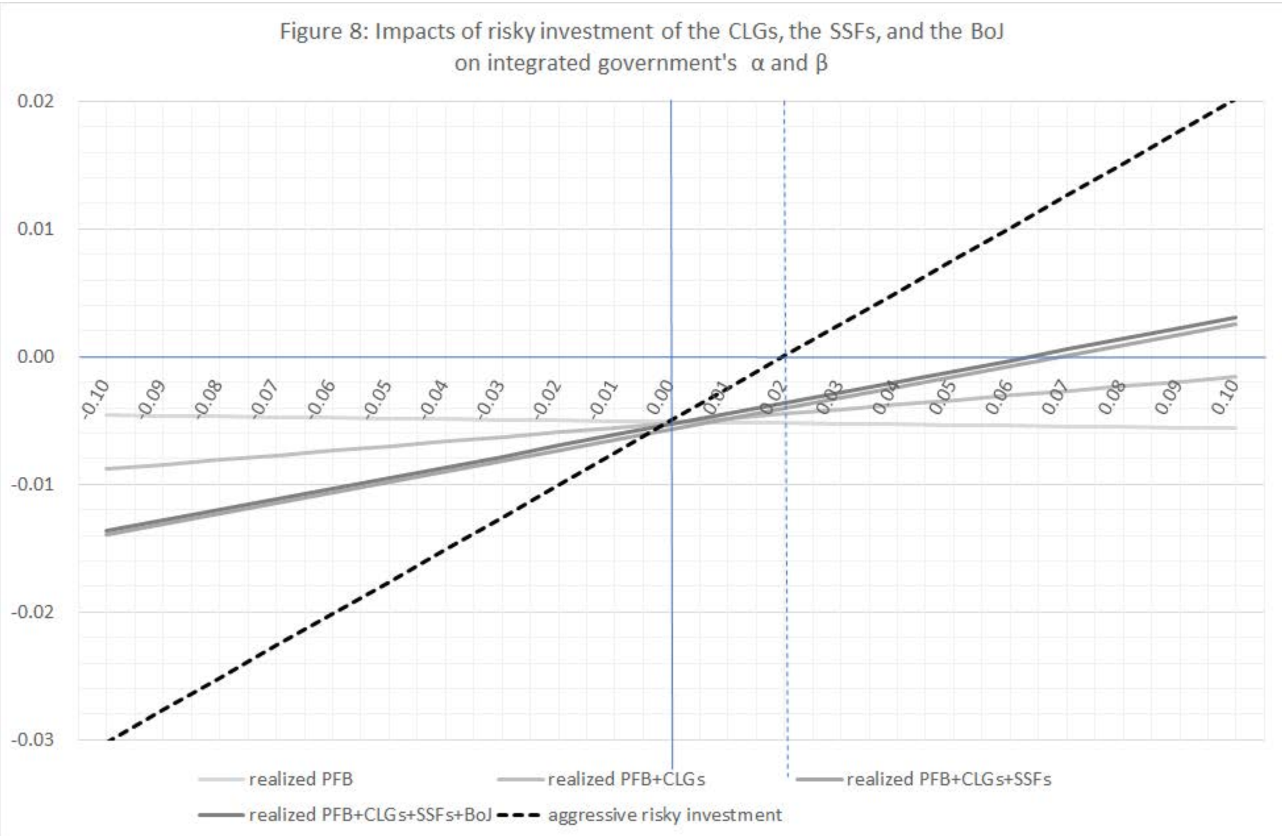


Figure 8: Impacts of risky investment of the CLGs, the SSFs, and the BoJ on integrated government's α and β
0.02
0.01
0.00
-0.01
-0.02
-0.03
-0.10
-0.09
-0.08
-0.07
-0.06
-0.05
-0.04
-0.03
-0.02
-0.01
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.10
realized PFB
realized PFB+CLGs
realized PFB+CLGs+SSFs
realized PFB+CLGs+SSFs+BoJ
aggressive risky investment