Fiscal multipliers in the most aged country: Empirical evidence and theoretical interpretation*

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Abstract

This study investigates how population aging impacts the effectiveness of a government spending shock. We estimate a panel VAR model with prefectural data in Japan, the world's fastest aging country and reveal that a government spending shock becomes less effective as the aging rate increases. Subsequently, we construct a New Keynesian model with workers and retirees, which can replicate our empirical findings. This highlights the role of the supply-side channel through which workers facing a liquidity constraint can benefit from increased disposable income, in generating the state-dependent effect of the government spending shock. Our theoretical finding may suggest that promoting labor market participation by elderly people could increase the effectiveness of a government spending shock amid a rapidly aging society.

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1 Introduction

Population aging is a growing issue faced by many developed countries. From the economic perspective, it can cause long-term problems, such as low economic growth and fiscal unsustainability. Indeed, many studies revealed the relationship between population aging and such problems in the growth model framework (e.g., Gonzalez-Eiras and Niepelt, 2012; Nishiyama, 2015; Hansen and İmrohoroğlu, 2016).

However, there is little research addressing this issue in the context of short-term economic fluctuations. Specifically, it is not clear how the aging structure affects the economic stimulus effect of fiscal policy shock, although population aging is likely to make fiscal policy management difficult.[1] In this study, we aim to clarify whether the effectiveness of a government spending shock on output changes depending on the economy's aging rate, both theoretically and empirically. This study contributes to the literature by presenting a desirable labor market structure that allows the government to maintain the effectiveness of a government spending shock amid rapid population aging.

To this end, we estimate a panel VAR model that includes government spending, tax revenue, and output across Japan's prefectures, as Japan is the world's most rapidly aging country. We illustrate that the progression of population aging is likely to dampen the effectiveness of a government spending shock measured by the fiscal multiplier. To interpret the empirical findings theoretically, we build a New Keynesian (NK) model with heterogeneous agents, in which we categorize households into three types: Ricardian workers, Non-Ricardian workers, and retirees. Our model replicates the empirical evidence well and carries the implication that the state-dependent effect of a government spending shock on the aging rate is mainly caused by the supply-side channel, for lack of which retirees cannot benefit from the increased disposable income arising from a government spending shock.

This study closely relates to that by Basso and Rachedi (forthcoming), who estimate

[1]Anderson et al. (2016) and Janiak and Santos-Monteiro (2016) are among the few to examine the connection between demographic structure and fiscal policy.

the fiscal multiplier in U.S. states depending on the aging structure and construct a life-cycle NK model to rationalize their empirical findings. Basso and Rachedi's (forthcoming) study is similar to ours in that we both reveal the negative effect of population aging on the fiscal multiplier. In this sense, our empirical analysis strengthens the findings in Basso and Rachedi (forthcoming), though our study differs from theirs in several regards. First, we derive the fiscal multiplier from the VAR model, while Basso and Rachedi (forthcoming) estimate a single equation to obtain a *local fiscal multiplier* in the manner of Nakamura and Steinsson (2014). The advantage of the VAR model over the single-equation model is that it provides the aggregate fiscal multiplier directly instead of having to derive the *local fiscal multiplier*. The disadvantage is its lower statistical accuracy, as the loss of degrees of freedom due to the inclusion of many endogenous variables leads to a wider error band for the estimates. To cope with this shortcoming, we incorporate a hierarchical structure into our panel VAR model. This method improves the statistical accuracy of estimated coefficients and thus underscores the differences across the demographic structures. In addition, the VAR analysis gives us the impulse responses of tax revenue, which we can use to calibrate the fiscal rule in the theoretical model. Second, regarding the theoretical model, we build on Yoshino and Miyamoto (2017), whose model has infinitely lived workers and retirees. In the absence of life-cycle agents, our model is more tractable than the one developed in Basso and Rachedi (forthcoming). Despite its simplicity, however, our theoretical model successfully traces the empirical responses qualitatively and quantitatively. Finally, and most importantly, we attempt to identify a source of state dependency in the efficacy of a government spending shock by altering the assumption of retirees' behavior. The results of this theoretical analysis imply that retirees' lack of access to the labor market is more relevant than is the lack of access to the financial market in generating the state-dependent effect on the aging situation.

The remainder of this paper is organized as follows. Section 2 describes our data, explains the empirical model, and presents the empirical evidence and the robustness of our main findings. In Section 3, we develop an NK model to rationalize our empirical findings. Section 4 examines the source of the state-dependent effect of the government spending shock by relaxing the assumption on retirees' behavior. Finally, Section 5 concludes the paper.

2 Empirical analysis

2.1 Data

The benchmark model is a Blanchard and Perotti (2002)-type three-variable fiscal VAR model that includes government spending (g_t), tax revenue (τ_t), and output (x_t) in real terms. We obtained the data on prefectural government spending and output from the *Annual Report on Prefectural Accounts* published by the Cabinet Office, Government of Japan. We retrieved the local tax revenue data for each prefecture from the *Nikkei NEEDS Financial Quest* database. The local tax consists mainly of the municipal tax and business tax collected by the local government, though this study does not cover all types of taxes. Since the original tax revenues for each prefecture are published in nominal terms, we deflate them by a prefecture-specific GDP deflator, which is also available in the *Annual Report on Prefectural Accounts*. We define government spending as the sum of government consumption and public investment. All data are at the annual frequency for the fiscal year. The sample period is from 1990 to 2014. [2]

To examine whether the efficacy of a government spending shock depends on the demographic structure, we divide the prefectures into two groups, low- and high-aging prefectures, depending on their aging ratio; that is, the ratio of the population over 65 to the total population. We construct this ratio using the population data published in *Population Estimates* by the Statistics Bureau of Japan, and then calculate its time average for each prefecture. Finally, we select the top (worst) 12 prefectures whose aging ratios are high (low).[3] The aging ratios differ across Japanese prefectures, as seen in

[2]The fiscal year in Japan starts on April 1 and ends the next March 31.

[3]Because Japan has 47 prefectures, the top and bottom 12 prefectures are approximately equal to the 1st and 3rd quartiles.

Figure 1, which displays the transition of the aging rate across prefectures using a box plot. Therefore, the Japanese prefectural data are suitable to our purpose of determining the efficacy of a government spending shock according to the aging situation, in the sense that the sizable heterogeneity among Japanese prefectures can highlight the difference in the value of the aging rate between the low- and high-aging groups.

[Figure 1 about here.]

The high aging prefectures include Akita (AK), Yamagata (YG), Nagano (NA), Wakayama (WA), Tottori (TT), Simane (SM), Yamaguchi (YA), Tokushima (TK), Ehime (EH), Kochi (KO), Oita (OI), and Kagoshima (KG). The low aging prefectures consist of Miyagi (MG), Ibaraki (IB), Tochigi (TC), Saitama (ST), Chiba (CB), Tokyo (TY), Kanagawa (KN), Aichi (AI), Shiga (SI), Osaka (OS), Hyogo (HG), and Okinawa (OK). As Figure 2 shows, the low-aging prefectures tend to be concentrated in metropolitan areas such as Tokyo and Osaka, while high-aging prefectures exist across the country. Here, we are not concerned about the location, but rather with whether or not these prefectures share common characteristics besides the demographic structure. To clarify that the state-dependent effect of a government spending shock, if any, is caused by the aging situation, it is necessary to prove that the aging rate in each prefecture is not related to other possible economic factors. As a representative example reported in Auerbach and Gorodnichenko (2012), the effectiveness of government spending shocks changes depending on economic conditions, such as a boom or recession. To this end, we compute the correlation between the aging rate and prefectural unemployment rate as a proxy for economic conditions. We find no significant correlation unless we include Okinawa prefecture. Okinawa, located in the south-most part of Japan, is an outlier with a considerably low aging rate and high unemployment rate, making the entire correlation significantly negative. Hence, we perform an additional estimation to check the robustness of the main results by excluding Okinawa from the low-aging prefectures.

[Figure 2 about here.]

2.2 Hierarchical panel VAR model with sign restrictions

We employ panel data on Japanese prefectures to examine whether the economic stimulus effect of increased government spending depends on the aging rate. As explained above, the sample period is from 1990 to 2014 with data at an annual frequency, and we divide the prefectures into two groups of 12 depending on the aging rate. One notable feature of this study is that we adopt a hierarchical panel VAR model to address the short sample period and the possible heterogeneity across the selected prefectures. As Canova and Pappa (2007) and Pappa (2009) emphasize, the hierarchical structure improves the quality of the estimates by exploiting unit-specific and cross-sectional information efficiently compared with the pooled estimators, which are likely to give us imprecise estimates in such circumstances.

For each unit i, we can formulate the three-variable VAR(p) model as

$$y_{it} = B_{i0} + B_{i1}y_{it-1} + \cdots + B_{ip}y_{it-p} + u_{it}, \tag{1}$$

where $y_{it} = [g_{it}, \tau_{it}, x_{it}]'$ is a 3×1 vector of endogenous variables, and u_{it} is a 3×1 vector of reduced-form residuals. We define $X_{it} = I_k \otimes \left[1, y'_{it-1}, \cdots, y'_{it-p}\right]$ and $\beta_i = \left[B'_{i0}, vec(B'_{i1}), \cdots, vec(B'_{ip})\right]'$, and rewrite the model as

$$y_{it} = X_{it}\beta_i + u_{it}, \tag{2}$$

where the *vec* operator creates a column vector from $B_{is}(s = 1, \cdots, p)$ by stacking the column vector of B_{is}, and $\otimes$ denotes the Kronecker product. We assume that we can represent the reduced-form residuals as a linear combination of structural shocks:

$$A_i u_{it} = e_{it}, \; e_{it} \sim N(0, D_i) \tag{3}$$

where e_{it} is a vector of structural shocks that can interpret economically and are mutually independent. Thus, the variance of each structural shock is located in a diagonal element of D_i. Note that the variances of structural shocks and contemporaneous matrix A_i are unit-specific, as are the VAR coefficients. Suppose that A_i is a lower triangular matrix, as in the form of

$$A_i = \begin{pmatrix} 1 & 0 & 0 \\ a_{21i} & 1 & 0 \\ a_{31i} & a_{32i} & 1 \end{pmatrix}. \tag{4}$$

Then, we can transform (3) into

$$u_{it} = Z_{it}\alpha_i + e_{it}, \tag{5}$$

where α_i is a stacked vector of the lower triangular elements in A_i, and Z_{it} takes the form of

$$Z_{it} = -\begin{pmatrix} 0 & 0 & 0 \\ u_{it}^1 & 0 & 0 \\ 0 & u_{it}^1 & u_{it}^2 \end{pmatrix}, \tag{6}$$

where u_{it}^l indicates the l-th element from the top of the vector of the reduced-form residual u_{it}. In summary, we can estimate the unit-specific coefficients β_i and contemporaneous relationship α_i from (2) and (5).

For the unit-specific coefficients β_i and α_i, we postulate the following hierarchical structure:

$$\beta_i = \bar{\beta} + \nu_i, \nu_i \sim N(0, \Sigma_\nu), \tag{7}$$

$$\alpha_i = \bar{\alpha} + \omega_i, \omega_i \sim N(0, \Sigma_\omega), \tag{8}$$

implying that each unit i's coefficients share a common prior distribution. Namely, each unit i's coefficients are random variables drawn from the common distribution. Using the Bayesian technique, we can calculate the conditional posterior distributions of β_i and α_i from the data and prior distributions (7) and (8). In addition, we can specify the conditional posterior distributions of $\bar{\beta}$ and $\bar{\alpha}$ by regarding individual draws of β_i and α_i as observations. Therefore, this mutual use of information between unit-specific and cross-sectional cyclically improves the accuracy of both estimates in the hierarchical model.

We identify the structural shocks using the sign restriction developed by Uhlig (2005). We adopt this method because, in contrast to Blanchard and Perotti (2002), who use quarterly data, it is reasonable to suppose that the fiscal variables respond to contemporaneous economic activity in annual data. The methodology, which imposes restrictions on the shape of the impulse responses, identifies the shocks by allowing for simultaneous responses among the variables. Therefore, we can elude the problem of endogeneity originating from the use of annual frequency data. The only shock of interest is government spending for this study; nevertheless, we follow Peersman (2005) to identify a full set of structural shocks, which can help us identify government spending shocks. Furthermore, Caldara and Kamps (2017) argue recently that the estimated impact of a government spending shock on output is determined by the extent to which the unconditional covariance between them is allocated as a contribution of the government spending shock. In this sense, it is preferable to identify shocks besides a government spending shock. The structural shocks identified here are government spending, tax revenue, and business cycle shocks.

Table 1 summarizes the sign restrictions employed in this study. Each row and column correspond to a shock and variable, respectively. We set the sign conditions to be basically consistent with the theoretical and empirical evidence documented in the existing literature. We impose all restrictions on the variables for two periods. We first assume that the government spending shock increases government spending for two years. Note that we impose no sign restrictions on the other variables to allow the data to speak for itself, as we are interested in the quantitative and qualitative effects of a government spending shock. Subsequently, we assume that output responds negatively to an exogenous positive tax revenue shock; that is, an unexpected increase in tax revenue. Mountford and Uhlig (2009) report a negative co-movement between tax revenue and output for the U.S. and Kuttner and Posen (2002) report the same for Japan. We assume that a favorable business cycle shock is a shock that increases output and tax revenue while decreasing government spending. We consider an increase in tax revenue due to a business cycle shock as an automatic response associated with an increase in the tax base. Conversely, this type of shock describes the situation in which a discretionary increase in government spending is implemented in response to a drop in tax revenue and output by a recession. The signs imposed on tax revenue and output can separate tax revenue shocks from business cycle shocks, and we can identify government spending shocks and business cycle shocks by the sign on the responses of government spending. However, we cannot distinguish between government spending shocks and tax revenue shocks sufficiently using the restrictions listed in Table 1 because there are no restrictions on government spending after a tax revenue shock, and vice versa. Hence, we impose a size restriction such that the impact response of government spending to government spending shocks in absolute terms is greater than that to tax revenue shocks. This condition assumes that the government spending shock best explains the forecast error variance in government spending during the impact period. We can consider this final restriction as the simplest version of Max Share identification proposed by Francis et al. (2014).

[Table 1 about here.]

In practice, identification using sign restrictions is implemented by an algorithm involving QR decomposition, as proposed by Rubio-Ramírez et al. (2010). After drawing a sample of β_i, A_i, and D_i for each unit i, we generate a matrix Q such as $W = QR$ by the QR decomposition, where Q is an orthogonal matrix (i.e., $QQ' = I$), R is an upper-triangular matrix, and W is a real square matrix. Moreover, we draw each column of W randomly from $N(0, I)$. Given the matrices of A_i, D_i, and Q, we can construct an alternative contemporaneous matrix $\tilde{A}_i$ and structural shocks $\tilde{\varepsilon}_{it}$ by transforming (3) into

$$\begin{aligned} u_{it} &= A_i^{-1} D_i^{1/2} \varepsilon_{it} \\ &= A_i^{-1} D_i^{1/2} Q Q' \varepsilon_{it} \\ &= \tilde{A}_i \tilde{\varepsilon}_{it}, \end{aligned} \tag{9}$$

where $\tilde{A}_i = A_i^{-1} D_i^{1/2} Q$, $\tilde{\varepsilon}_{it} = Q' \varepsilon_{it}$, and the new structural shock ε_t follows a standard normal distribution. We repeatedly draw a matrix Q until the impulse response computed from β_i and $\tilde{A}_i$ satisfy the sign restrictions in Table 1.

Following Canova and Pappa (2007) and Pappa (2009), we assume that the impulse response function also has a hierarchical structure. We denote the stacked vector of each unit i's impulse responses of the variables to the government spending shock as ψ_i. Representing the maximum horizon of impulse responses as J, ψ_i is a $3J \times 1$ vector storing the respective responses of government spending, tax revenue, and output until horizon J, in this order. Then, we impose the unit invariant specification on ψ_i, given as

$$\psi_i = \bar{\psi} + \eta_i, \eta_i \sim N(0, \Sigma_\eta), \tag{10}$$

where $\bar{\psi}$ is the "typical" responses according to the terminology in Pappa (2009), Σ_η is a diagonal matrix, and $(\Sigma_\eta)_j, j = 0, 1, \cdots, J$, which corresponds to the variance of the responses at horizon j, is set to $\frac{0.2}{j}$ exogenously, as in Canova and Pappa (2007).

Finally, we discuss the prior distributions adopted in this study. As discussed above, the prior distributions for β_i and α_i are

$$\beta_i \sim N(\bar{\beta}, \Sigma_\nu) \quad \alpha_i \sim N(\bar{\alpha}, \Sigma_\omega), \tag{11}$$

as described in (7) and (8). Note that the hyperparameters in (11) also generate their conditional posterior distributions. While we assume diffuse priors on $\bar{\beta}$ and $\bar{\alpha}$, we set the priors for Σ_ν and Σ_ω to be an inverse Wishart distribution:

$$\Sigma_\nu \sim \mathcal{IW}(100, 0.01I), \quad \Sigma_\nu \sim \mathcal{IW}(100, 0.01I), \tag{12}$$

where $\mathcal{IW}$ represents the inverse Wishart distribution. We assume that the prior distribution for the inverse of the l-th diagonal element of D_i, denoted by δ_i^l, follows a gamma distribution, as follows:

$$\left(\delta_i^l\right)^{-1} \sim \mathcal{G}\left(\frac{100}{2}, \frac{0.01}{2}\right), \tag{13}$$

where $\mathcal{G}$ also indicates a gamma distribution. Similar to $\bar{\beta}$ and $\bar{\alpha}$, we set the prior for impulse response $\bar{\psi}$ to be diffuse priors.

2.3 Empirical results

All data in the VAR system are at the log level of the real per-capita values. The system contains a constant and linear trend as a deterministic term and we set the lag length to two as we use annual frequency data. We conduct the estimation using the Gibbs sampler of the Bayesian Markov Chain Monte Carlo (MCMC) method, iterating $60,000$ draws of each parameter from the conditional posterior distributions and discarding the first $10,000$ draws as a burn-in. Then, we exploit every 10-th draw for the inference to alleviate the autocorrelation among each sequential draw. Moreover, we save only the draws in which the roots of the VAR coefficients are inside the unit circle to ensure the stationarity of the VAR system.

Figure 3 illustrates the responses of (a) government spending, (b) tax revenue, (c) output, and (d) the accumulated fiscal multiplier. We denote the median responses obtained from the low- and high-aging prefectures by solid lines and dash-dotted lines with circles, with the associated 68% credible intervals depicted by shaded areas and dash-dotted lines. We normalize the scale of all responses to be interpreted as yens. For example, an increase in government spending of 1 yen leads to an increase in output of 1.5 yen in the impact period in low-aging prefectures. We compute the J-period-ahead accumulated fiscal multiplier as

$$\text{Accum. fiscal multiplier}_J = \frac{\sum_{j=0}^{J} IR_j(x)}{\sum_{j=0}^{J} IR_j(g)} \times \frac{\bar{x}}{\bar{g}}, \tag{14}$$

where $IR_j(\cdot)$ denotes the impulse response of the variable in the parentheses at horizon j, and $\bar{x}$ and $\bar{g}$ represent the means of output and government spending in each group, taking the average for both the time-series and cross-sectional dimensions. Fiscal multipliers allow us to quantitatively compare the effect of a government spending shock on output, even if the dynamics of government spending differ between the two groups.

[Figure 3 about here.]

We first observe from Figure 3 that output exhibits different dynamics across the two groups to the same 1-yen increase in government spending. In particular, the impact response of output in low-aging prefectures is at least 1.5 times that in high-aging prefectures, suggesting that the effect of government spending shocks changes depending on the demographic structure. When assessing the difference in the effects across the groups over the impact period, it is appropriate to examine the fiscal multiplier instead of the output response because the path of government spending is also different across the two groups, as Figure 3 (a) shows. The fiscal multiplier, derived from (14), measures how much output increases for 1 yen of government spending for each horizon based on the cumulative response of each variable so far, allowing us to directly compare the magnitude of a government spending shock on output across the groups by unifying the size of government spending. Figure 3(d) shows that the response of the multiplier in low-aging prefectures is always located above the one in high-aging prefectures without overlap between the confidence bands, demonstrating that population aging diminishes the stimulus effect of a government spending shock on output significantly. In addition, the fact that the size of the multiplier in the high-aging prefecture fluctuates around one suggests that there are a little or no "multiplier effects" under the condition of progressive population aging. On the other hand, the multiplier in low-aging prefectures is larger than those derived from the standard neoclassical model (e.g., Aiyagari et al., 1992; Baxter and King, 1993), matching those derived from the "ultra" NK model (e.g., Galí et al., 2007). This motivates us to build on the Galí et al. (2007)-type NK model in the theoretical part below to rationalize our empirical findings.

Finally, we discuss the fiscal stance in Japan by examining the response of tax revenue in Figure 3 (b). The path of the tax response also changes depending on the degree of population aging, which traces that of output proportionally. Additionally, we should stress that the scales of the tax response are considerably modest compared with that of government spending, regardless of the situation. Although we do not cover all types of tax revenue, our estimates indicate that the increase in tax revenue is only about 10% of the increase in government spending, suggesting that government spending in Japan is mostly financed by debt issuance rather than tax increases. Prior studies often point out this lack of fiscal discipline (Doi et al., 2011). We use this VAR result to calibrate the structural parameters in the theoretical model below.

In sum, our empirical findings strongly support the state-dependent effect of a government spending shock on the aging structure in Japan. More specifically, in line with Basso and Rachedi (forthcoming), the fiscal multiplier in low-aging prefectures is significantly higher than that in high-aging prefectures. We next address the factors that cause state dependency, as we observed in the empirical analysis. To answer this question, we construct a dynamic stochastic general equilibrium (DSGE) model in the next section to derive a structural interpretation of the empirical evidence.

2.4 Robustness check: Excluding the outlier

Before we construct a theoretical model that can replicate the state-dependent effect of a government spending shock on the aging ratio observed in the data, we perform a robustness check of the main empirical findings. As mentioned above, Okinawa prefecture, which is a low-aging prefecture, is an outlier in terms of the aging rate and unemployment rate; it has the lowest aging rate and the highest unemployment rate in Japan. This strict negative relationship generates a significant negative correlation between the aging rate and the unemployment rate for all prefectural data. This is problematic because the selected prefectures are likely to be categorized by economic conditions rather than aging, meaning that it remains possible that the state-dependency is caused by factors other than population aging. Fortunately, however, this correlation is insignificant for the data except for Okinawa prefecture. Hence, we now re-estimate our empirical model for low-aging prefectures without Okinawa's data.

In Figure 4, we plot the median responses of the variables derived from the sample excluding Okinawa prefecture (denoted by dashed lines), together with the median responses and credible intervals derived from the benchmark estimates. At a glance, we find that the benchmark findings are robust because the responses derived from the robustness check almost follow the ones from the benchmark, leading us to conclude that population aging, not economic conditions, is a main factor that influences the effectiveness of a government spending shock in our present analysis. Based on the empirical findings so far, we build a DSGE model in which the aging ratio affects the stimulus effects of a government spending shock on output.

[Figure 4 about here.]

3 Theoretical model

To rationalize the empirical evidence using a theoretical model, we construct a simple NK model that contains a retiree as the counterpart of an elderly person in the empirical analysis. Our theoretical model is similar to that in Yoshino and Miyamoto (2017), except for the presence of non-Ricardian households and wage unions, which are keys to replicate the large fiscal multipliers observed in low-aging regions.

3.1 Households

We divide households into three types: Ricardian workers, Non-Ricardian workers, and retirees. The difference between workers and retirees is whether the households provide labor services. While workers earn wages as compensation for labor supply, retirees receive social security benefits from the government. We assume that a fraction ζ of the population consists of retirees, and the remaining $1-\zeta$ consists of workers. In addition, we categorize workers as Ricardian or and Non-Ricardian. Non-Ricardians cannot access the bond market, but Ricardians can. In other words, Ricardians can smooth their consumption by bond holdings, while Non-Ricardians face liquidity constraints and thus consume all their disposable income each period. Among the workers, we assume that the fraction ξ consists of non-Ricardians and the remaining population consists of Ricardians. We also assume that retirees are unable to access the bond market and thus behave as Non-Ricardians.

3.1.1 Ricardian workers

Ricardian workers derive utility from consumption $c_{w,t}^R$ and government spending g_t, while receiving disutility from labor supply $h_{w,t}$. The lifetime utility function is

$$E_0 \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho}\right)^t \left\{ \frac{1}{1-\sigma} \left[\left\{ \omega c_{w,t}^{R\,\frac{\eta-1}{\eta}} + (1-\omega) g_t^{\frac{\eta-1}{\eta}} \right\}^{\frac{\eta}{\eta-1}} \right]^{1-\sigma} - \frac{h_{w,t}^{1+\mu}}{1+\mu} \right\}, \tag{15}$$

where the parameters $\rho \geq 0$, $\sigma \geq 0$, and $\mu \geq 0$ represent the rate of time preference, risk aversion, and the inverse of the Frisch labor elasticity, respectively. In addition, the parameter η is the elasticity of substitution between consumption and government spending, and the weight $\omega \in (0,1)$ governs the effect of government spending on utility. We follow Galí et al. (2007) and assume an imperfectly competitive labor market, where the labor unions set real wages, firms determine labor demand under the given wages, and workers supply their labor to fulfill the labor demand from firms. Moreover, we assume that labor demand is the same across all types of workers (hence omitting the superscript

R from the labor supply). Therefore, Ricardian workers choose only consumption to maximize (15) subject to the budget constraint

$$P_t c_{w,t}^R + \frac{B_{w,t}^R}{R_t} = P_t w_t h_{w,t} + B_{w,t-1}^R + P_t d_{w,t}^R - P_t \tau_{w,t}^R, \tag{16}$$

where P_t is the final good price, $B_{w,t}^R$ is the nominal government bond, w_t is real wages, $d_{w,t}^R$ is the real dividend Ricardian workers receive, $\tau_{w,t}^R$ is the lump-sum tax imposed on Ricardian workers, and R_t is the gross nominal interest rate. We can solve the first-order conditions of the Ricardian workers' problem as

$$\lambda_t^R = \left(\omega {c_{w,t}^R}^{\frac{\eta-1}{\eta}} + (1-\omega) g_t^{\frac{\eta-1}{\eta}}\right)^{\frac{1-\sigma\eta}{\eta-1}} \omega {c_{w,t}^R}^{-\frac{1}{\eta}}, \tag{17}$$

$$\lambda_t^R = \frac{1}{1+\rho} E_t \frac{R_t}{\Pi_{t+1}} \lambda_{t+1}^R, \tag{18}$$

where $\Pi_{t+1} \equiv P_{t+1}/P_t$ is the gross inflation rate and λ_t^R is the Lagrange multiplier on the Ricardian workers' budget constraint, representing the marginal utility of consumption.

3.1.2 Non-Ricardian workers

The period utility function of non-Ricardian workers is also

$$\frac{1}{1-\sigma}\left[\left\{\omega {c_{w,t}^N}^{\frac{\eta-1}{\eta}} + (1-\omega) g_t^{\frac{\eta-1}{\eta}}\right\}^{\frac{\eta}{\eta-1}}\right]^{1-\sigma} - \frac{h_{w,t}^{1+\mu}}{1+\mu}, \tag{19}$$

where $c_{w,t}^N$ is the consumption of non-Ricardian workers. We assume that non-Ricardian workers simply consume all their current disposable income every period because they lack access to the bond market. Therefore, we can represent their consumption as

$$c_{w,t}^N = w_t h_{w,t} - \tau_{w,t}^N, \tag{20}$$

where $\tau_{w,t}^N$ is the lump-sum tax paid by non-Ricardian workers, which is set to equate the steady-state consumption between Ricardian and non-Ricardian workers.

3.1.3 Retirees

Retirees consume social security benefits in each period. Thus, we can describe their behavior as

$$c_{r,t} = s_r, \tag{21}$$

where $c_{r,t}$ is the retiree's consumption and s_r is the retiree's social security benefit. Here, we assume that the social security benefit is time-invariant. Similar to the lump-sum tax imposed on workers, we set the steady-state values of the social security benefit for retirees to be equal to the steady-state consumption of all types of households.

3.2 Firms

The production sector consists of two types of firms: monopolistically competitive intermediate goods firms whose goods are differentiated by firm and a perfectly competitive final good producer who transforms intermediate goods into final goods using constant elasticity of substitution technology.

3.2.1 The final good firm

A perfectly competitive final good firm bundles a continuum of differentiated intermediate goods $y_{j,t}, j \in [0,1]$ into final good y_t using constant returns technology:

$$y_t = \left(\int_0^1 y_{j,t}^{\frac{\varepsilon_p - 1}{\varepsilon_p}} dj \right)^{\frac{\varepsilon_p}{\varepsilon_p - 1}}, \tag{22}$$

where ε_p is the degree of substitution between the different intermediate goods. Given the final good price P_t and the prices for intermediate goods $P_{j,t}$, the profit maximization problem for the final good firm yields the following demand function for intermediate goods:

$$y_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\varepsilon_p} y_t. \tag{23}$$

Substituting (23) into (22), we obtain the final good pricing rule:

$$P_t = \left(\int_0^1 P_{j,t}^{1-\varepsilon_p} dj\right)^{\frac{1}{1-\varepsilon_p}}. \tag{24}$$

3.2.2 Intermediate goods firms

We assume a linear function for the production technology of the intermediate goods firms:

$$y_{j,t} = h_{j,t}, \tag{25}$$

where $h_{j,t}$ is the labor hired by firm j. Then, the marginal costs mc_t of the firm are equal to the real wage:

$$mc_t = w_t. \tag{26}$$

We assume that intermediate goods firms set the price for their goods to maximize their profit under Calvo (1983)-type price stickiness. Let $P_{j,t}^*$ be the optimal price that the firm can choose in period t; then, we can write the profit maximization problem as

$$\max_{P_{j,t}^*} E_t \sum_{i=0}^{\infty} \left(\frac{\theta}{1+\rho}\right)^i \left[P_{j,t}^* - P_{t+i} mc_{t+i}\right] y_{j,t+i}, \tag{27}$$

subject to the demand function (23). Here, the parameter θ denotes the probability that the price cannot be reset at period t. This maximization problem yields the following optimal price:

$$P_{j,t}^{*} = \frac{\varepsilon_p}{\varepsilon_p - 1} \frac{E_t \sum_{i=0}^{\infty} \left(\frac{\theta}{1+\rho}\right)^i P_{t+i} y_{j,t+i} mc_{t+i}}{E_t \sum_{i=0}^{\infty} \left(\frac{\theta}{1+\rho}\right)^i y_{j,t+i}}. \tag{28}$$

Because all intermediate goods firms that can reoptimize their price in period t set the same markup $\varepsilon_p/(\varepsilon_p - 1)$, the optimal price is the same for all $1-\theta$ firms that can adjust their prices (i.e., $P_{j,t}^{*} = P_t^{*}$). On the other hand, we assume that the remaining θ fraction of the firms keep their prices set at period $t-1$. Therefore, under the final good pricing rule (24), the evolution of the aggregate price is

$$P_t^{1-\varepsilon_p} = \theta P_{t-1}^{1-\varepsilon_p} + (1-\theta)(P_t^{*})^{1-\varepsilon_p}. \tag{29}$$

3.3 Wage union

As mentioned above, we follow Galí et al. (2007) and introduce an imperfectly competitive labor structure, where labor unions exist for each type of differentiated labor input indexed by $l \in [0,1]$ and they determine real wages collectively. In addition, we assume that the Ricardian and Non-Ricardian workers are uniformly distributed across unions. A perfectly competitive labor bundler provides effective labor $h_{j,t}$ to intermediate good firm j by bundling the differentiated labor input $h_{j,t}(l)$ according to

$$h_{j,t} = \left(\int_0^1 h_{j,t}(l)^{\frac{\varepsilon_w - 1}{\varepsilon_w}} dl \right)^{\frac{\varepsilon_w}{\varepsilon_w - 1}}, \tag{30}$$

where ε_w is the degree of substitution between different labor inputs. Parallel to the problem for the final good producer, the labor bundler's optimal problem yields the labor demand function:

$$h_t(l) = \left(\frac{w_t(l)}{w_t} \right)^{-\varepsilon_w} h_t. \tag{31}$$

The labor union l sets the real wage to maximize the following objective function:

$$(1-\xi)\lambda_t^R[w_t(l)h_t(l)] + \xi\lambda_t^N[w_t(l)h_t(l)] - \frac{h_t(l)^{1+\mu}}{1+\mu} \tag{32}$$

subject to (31). Note that the labor income for each type of worker is weighted by their marginal utility of consumption, denoted by λ_t^R and λ_t^N, respectively.[4] Solving this problem, we obtain the optimal wage schedule:

$$\left(\frac{(1-\xi)\lambda_t^R + \xi\lambda_t^N}{h_t^{\mu}}\right) w_t = \frac{\varepsilon_w}{\varepsilon_w - 1}, \tag{33}$$

where we omit the l index in symmetric equilibrium. This equation states that the real wage is set by adding the markup $\varepsilon_w/(\varepsilon_w - 1)$ to the (weighted sum of) the marginal rate of substitution.

3.4 Fiscal and monetary authorities

The government budget constraint in nominal terms is

$$P_t\tau_t + \frac{B_t}{R_t} = P_t g_t + B_{t-1} + P_t s, \tag{34}$$

where τ_t is the aggregate lump-sum tax. By defining $\hat{\tau}_t \equiv (\tau_t - \tau)/y$, $\hat{g}_t \equiv (g_t - g)/y$, and $\hat{b}_t \equiv (b_t - b)/y$, we adopt the following tax rule:

$$\hat{\tau}_t = \phi_b \hat{b}_{t-1} + \phi_g \hat{g}_t, \tag{35}$$

where any variables without a time subscript indicate the steady-state values of the corresponding variables. This form of tax rule allows for debt financing. We assume that

[4]From the utility function (19), the marginal utility of consumption for Non-Ricardians is

$$\lambda_t^N = \left(\omega {c_{w,t}^N}^{\frac{\eta-1}{\eta}} + (1-\omega) g_t^{\frac{\eta-1}{\eta}}\right)^{\frac{1-\sigma\eta}{\eta-1}} \omega {c_{w,t}^N}^{-\frac{1}{\eta}},$$

taking the same form as for Ricardians.

the dynamics of government spending follow the AR(1) process:

$$\hat{g}_t = \rho_g \hat{g}_t + u_t^g, \tag{36}$$

where $0 < \rho_g < 1$ and u_t^g is an exogenous innovation on government spending with $u_t^g \sim N(0, \sigma_g^2)$.

Finally, we assume that the monetary policy follows a simple Taylor rule:

$$\frac{R_t}{R} = \left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi}. \tag{37}$$

3.5 Aggregation

Three types of households exist in the model economy: Ricardian workers, Non-Ricardian workers, and retirees. The shares of these households are $(1-\xi)(1-\zeta)$, $\xi(1-\zeta)$, and ζ, respectively. Therefore, the aggregate consumption c_t is

$$c_t = (1-\xi)(1-\zeta)c_{w,t}^R + \xi(1-\zeta)c_{w,t}^N + \zeta c_{r,t}. \tag{38}$$

Only workers provide the labor input, and it is the same across the types of workers. Hence, the aggregate labor input h_t is

$$h_t = (1-\zeta)h_{w,t}. \tag{39}$$

Similarly, workers pay a lump-sum tax. Thus, we have

$$\tau_t = (1-\zeta)\left\{(1-\lambda)\tau_{w,t}^R + \lambda\tau_{w,t}^N\right\}, \tag{40}$$

where the lump-sum tax differs depending on the type of worker. In contrast, social security benefits are distributed to the retiree:

$$s = \zeta s_r. \tag{41}$$

Finally, only Ricardians receive dividends and hold government bonds, and thus we represent the aggregate dividends and bonds as

$$d_t = (1-\lambda)(1-\zeta)d^R_{w,t}, \tag{42}$$

$$b_t = (1-\lambda)(1-\zeta)b^R_{w,t}. \tag{43}$$

3.6 Market clearing

Equilibrium in the labor market requires

$$h_t = \int_0^1 h_{j,t}dj, \tag{44}$$

implying that the supply of labor input equals its demand. The final goods market clearing condition is

$$y_t = c_t + g_t \tag{45}$$

because we assume a no capital closed economy in this study.

3.7 Calibration

To quantify the dynamic responses to a government spending shock, we calibrate the model to the Japanese economy for the sample period from 1990 to 2014. Table 2 summarizes the calibrated values for the parameters. The purpose of this exercise is to examine theoretically whether the effect of a government spending shock changes depending on the fraction of retirees, as it does in the empirical analysis. We derive the rational expectations equilibrium from a system of stochastic differential equations composed of the log-linearized equilibrium conditions, which we provide in Appendix A.1.

We set the model period to one year to match the frequency in the empirical analysis. First, we set the rate of time preference to 0.01 such that the annual real interest rate equals 1%, which approximately corresponds to the time average of the ex-post real interest rate for the sample period.[5] Based on the sample average in the data, we set the steady-state values for the debt-to-output ratio at $\gamma_b = 1.5$. Similarly, the steady-state share of government spending in output is $\gamma_g = 0.3$ from the data, implying that the share of consumption in the output of $\gamma_c = 0.7$.

Regarding the parameters in the utility function, we set the risk aversion to $\sigma = 1.5$ and the inverse of labor supply elasticity to $\mu = 1$. We choose these parameters as they are consistent with the micro evidence documented by Abe et al. (2007) and Kuroda and Yamamoto (2008), respectively. Moreover, we follow Bouakez and Rebei (2007) and set $\omega = 0.8$ and $\eta = 0.3$ such that private consumption and government spending are complements. Okubo (2003) also provides evidence supporting the complementarity between them in Japan. As for price stickiness, we set the Calvo parameter to $\theta = 0.55$ to accord with Watanabe and Watanabe's (2018) finding that the annual rate of price changes for 55% of items that constitute the Japanese CPI is almost zero. The fraction of Non-Ricardians is $\xi = 0.13$, following Hara et al. (2016), who calculates the share of hand-to-mouth households, including wealthy hand-to-mouth, in Japan using national household survey data.

Following Galí et al. (2007), we calibrate the parameters in the tax rule and dynamics of a government spending shock to be consistent with our VAR evidence in Figure 3. Since the dynamics of the fiscal variables seem to differ depending on the demographic structure, we set some parameter values for each case. Specifically, we set the elasticity of

[5] We calculate the ex-post real interest rate by subtracting the inflation rate (CPI; all items) from the uncollateralized overnight call rate.

tax revenue to government spending ϕ_g to 0.08 in the low-aging cases and 0.04 in the high-aging cases to match the respective tax responses during the impact period. On the other hand, our empirical result demonstrates that tax revenues do not increase sufficiently in response to an increase in government spending in both cases, implying that fiscal policy in Japan might be unsustainable. Doi et al. (2011) also point out this implication in their estimate of the Japanese tax rule using a two-state Markov switching model. Doi et al. (2011) find an insignificant coefficient estimate on the debt-to-GDP ratio regardless the state, suggesting that the fiscal stance in Japan is "active" according to Leeper's (1991) terminology. However, we must ensure fiscal sustainability in the model economy to determine a unique equilibrium. Thus, we unify $\phi_b = 0.01$ in both aging cases, which is the smallest value that guarantees a unique equilibrium. Similarly, we set the persistence of government spending to $\rho_g = 0.6$ in the low-aging state and $\rho_g = 0.75$ in the high-aging state such that the theoretical responses of government spending at five periods after the shock correspond to the empirical responses. Finally, we follow Miyazawa (2011) and set $\phi_\pi = 1.2$, which also satisfies the Taylor principle.

[Table 2 about here.]

3.8 Theoretical responses

We calculate the impulse responses in two aging cases by setting $\zeta = 0.2$ in a low-aging economy and $\zeta = 0.35$ in a high-aging economy, which are approximately equivalent to the aging rate in the lowest and highest prefectures in 2014. Figure 5 displays the dynamic responses of (a) government spending, (b) tax revenues, (c) output, and (d) the calculated cumulative fiscal multiplier to a positive government spending shock. In line with the empirically determined responses shown in Figure 3, we plot the responses in low- and high-aging economies using solid lines and dashed-dotted lines with circles, respectively. Note that we normalize the scale of all responses in Figure 5 by steady-state output because we define the theoretical responses of government spending and tax revenue as a deviation from the steady state and normalized by steady-state output as in Galí et al. (2007), allowing us to directly compare the theoretical and empirical responses.

[Figure 5 about here.]

Overall, the theoretical responses closely follow the empirical responses in quantitative and qualitative terms, although we cannot reproduce some hump-shaped responses due to the simplicity of the model. Particularly, it is worth noting that our model perfectly replicates the positional relationships between the responses such that the fiscal multiplier in a low-aging economy exceeds that in a high aging economy. We focus on the output responses to elucidate the mechanism by which our model generates the state dependency of the fiscal multiplier on the aging ratio. A glance at Figure 5 (c) reveals that in both aging cases, a one-unit increase in government spending leads to a more than one-unit increase in output at the impact period. From the market-clearing condition in (45), we can attribute the gap in the responses between output and government spending to an increase in consumption. The presence of non-Ricardian workers and wage unions causes an increase in consumption, in contrast to a standard DSGE model, in which private consumption responds negatively to a positive government spending shock due to a negative wealth effect. As with Galí et al. (2007), consumption by Non-Ricardians greatly increases owing to the postponement of a tax increase via debt financing and an increase in real wages set by wage unions. A further important point is that, unlike Galí et al. (2007), consumption by Ricardians also responds positively to an increase in government spending thanks to a complementarity between them. Accordingly, while retirees cannot change their consumption at all, both types of workers increase consumption after a positive government spending shock, resulting in an increase in output. As the share of retirees declines, the positive movement of workers' consumption is increasingly reflected in the aggregate consumption. Hence, it follows that the stimulus effect of a government spending shock on output becomes higher in a low-aging-rate economy. This is a key mechanism for generating the state dependency observed in our theoretical model. Additionally, the reversal of output responses after period 2 is due to the difference in the persistence of government spending. As in the empirical findings, the fiscal multipliers in low-aging economies continue to lie above those in high-aging economies, even though the position of the output responses is reversed.

Moreover, the degree of population aging influences the size of the fiscal multiplier through our specific labor market structure. Note that output is equivalent to labor supply because we adopt no capital and linear production function assumption in our model, and real wages move in the same direction as labor supply by (32). Then, an increase in output causes a rise in both labor supply and real wages, leading to a rise in current disposable income and consumption by Non-Ricardians. Therefore, the labor market channel, through which the consumption by Non-Ricardians expands cyclically, plays an additional role in enhancing the effect of a government spending shock on output when the share of retirees is low.

The channels that generate the state dependency are similar to those in Basso and Rachedi (forthcoming) in that the results emphasize the positive responses of consumption and labor supply by a specific type of household to a government spending shock. In Basso and Rachedi's (forthcoming) model, young workers who have a high labor supply elasticity and high marginal propensity to consume (as they face liquidity constraints) are a major factor that alters the fiscal multipliers depending on demographics. In terms of liquidity constraints and a high marginal propensity to consume, young workers could be regarded as a counterpart to the non-Ricardian workers who greatly increase their consumption and labor supply in response to a government spending shock. Similar to the behavior of non-Ricardian workers in this study, young workers boost their labor supply and consumption in response to an increase in government spending, leading to a significant increase in aggregate output when the share of young workers is high. The negative wealth effect in our model does not increase the labor supply due to the existence of labor unions. Except for the origin of the increased labor supply, our theoretical model shares almost the same mechanism as the one in Basso and Rachedi (forthcoming).

Consequently, it follows from our theoretical exercise that population aging reduces fiscal policy effectiveness in terms of both the demand and supply sides. From the demand side, a rise in the share of retirees, which is equivalent to decreasing the share of workers, decreases the positive response of aggregate consumption because retirees' consumption is always unchanged, resulting in a lower output response. From the supply side, retirees have no labor supply by definition, so they cannot benefit from an increase in disposable income when wage unions set a higher wage in line with the rise in labor demand involved in the surges of government demand. Consequently, the stimulus effect on consumption becomes more modest as population aging progresses. To summarize, we can theoretically interpret the state-dependent effect of a government spending shock on the aging rate observed in the empirical analysis as stemming from the behavior of retirees, whose consumption and labor supply do not react to government spending shocks at all.

4 Discussion

As retirees have no access to the labor and financial markets, a government spending shock has no effect on their behavior in the benchmark theoretical model. However, these two non-access assumptions are essential factors to characterize retirees in the model economy, though it is not necessarily reasonable in practice. In particular, the Japanese official household surveys, such as the *Family Income and Expenditure Survey* report that elderly people in Japan have abundant financial assets, indicating the possibility that retirees can access the financial market and thus smooth their consumption. In this section, we examine how our theoretical outcome changes when retirees can access the financial market. We should stress that this analysis aims not only to bring the model economy closer to the real economy, but also to clarify whether the aforementioned demand- or supply-side factor contributes more to generating the state dependency of a government spending shock. Since retirees still have no access to the labor market and thus cannot benefit from an increase in disposable income in the model we develop in this section, comparing the results from this model and the benchmark will highlight the role of demand-side factors; that is, consumption fluctuations. Hence, we can conclude that demand-side factors (i.e., no consumption smoothing) are more relevant than supply-side factors (i.e., no labor supply) are in generating the state-dependent effect of a government spending shock if it disappears in the current model. Conversely, if it does not disappear, then it follows that the supply-side channel, through which workers' consumption expands cyclically, is the essential factor generating the state dependency.

We refer to retirees who can access the bond market and smooth their consumption by bond holdings as Ricardian retirees. Since retirees do not work, though they are Ricardians, their lifetime utility function is

$$E_0 \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho} \right)^t \left\{ \frac{1}{1-\sigma} \left[\left\{ \omega c_{r,t}^{R\,\frac{\eta-1}{\eta}} + (1-\omega) g_t^{\frac{\eta-1}{\eta}} \right\}^{\frac{\eta}{\eta-1}} \right]^{1-\sigma} \right\}, \tag{46}$$

where $c_{r,t}^R$ denotes the consumption of Ricardian retirees. This lifetime utility function is maximized subject to the budget constraint

$$P_t c_{r,t}^R + \frac{B_{r,t}^R}{R_t} = B_{r,t-1}^R + P_t d_{r,t}^R + P_t s_r^R. \tag{47}$$

Ricardian retirees receive social security benefits, dividends, and repayments from bond holdings carried over from an earlier period, which they allocate to consumption and purchase of new bonds. Although we assume that the social security benefit is time-invariant, unlike in the benchmark model, Ricardian retirees can now change their consumption intertemporally through bond holdings. The optimization problem yields the marginal utility of consumption and the Euler equation for Ricardian retirees with the same form as the Ricardian workers' equations in (17) and (18). Namely, it follows that the Ricardians' optimal intertemporal consumption condition is the same regardless of whether they are workers or retirees, leading to the same responses of consumption to a government spending shock in both types of Ricardians.[6]

Figure 6 shows the impulse responses derived from the model in which retirees behave as Ricardians. The introduction of Ricardian retirees slightly magnifies the responses of output and the fiscal multiplier compared to Figure 5 because of the complementarity between consumption and government spending by which Ricardian retirees increase their consumption in response to a government spending shock. However, the gap remains in the responses across the different aging ratios, even if we allow retirees to access the financial market, suggesting that the demand-side factor (retirees' lack of access to the financial market) alone does not explain the state-dependent effect of a government spending shock on the share of retirees. Therefore, we deduce from our theoretical exercise that the supply-side factor; that is, retirees' lack of access to the labor market, plays an important role in generating the situation in which population aging diminishes the effectiveness of a government spending shock. In other words, we can attribute the state-dependent effect of a government spending shock mainly to retirees' receiving no benefit from higher wages triggered by an increase in government spending as they supply no labor. Importantly, this theoretical finding have the practical implication that the stimulus effect of a government spending shock is likely to increase if elderly people are encouraged to participate in the labor market.

[Figure 6 about here.]

[6] In a practical manner, the introduction of Ricardian retirees modifies the system of the log-linearized rational expectation equilibrium described in Appendix A.1 as follows. First, we add an equation to indicate that the deviation of Ricardian retirees' consumption is equal to that of Ricardian workers:

$$\hat{c}^R_{r,t} = \hat{c}^R_{w,t}.$$

Second, the aggregate consumption equation becomes

$$\hat{c}_t = (1-\xi)(1-\zeta)\hat{c}^R_{w,t} + \xi(1-\zeta)\hat{c}^N_{w,t} + \zeta\hat{c}^R_{r,t},$$

such that Ricardian workers' consumption now fluctuates in response to the shock.

5 Conclusion

In this study, we examined how population aging affects the economic stimulus effect of a government spending shock. Specifically, we explored whether the effectiveness of the government spending shock on output changes depending on the demographic structure. Given the relatively small amount of research investigating the relationship between population aging and fiscal policy effectiveness in terms of short-run business fluctuations, the present study addresses this issue from both the theoretical and empirical perspectives. We first reveal empirically that the government spending multiplier would diminish as population aging progresses. Then, we develop an NK-DSGE model with heterogeneous agents to explain the empirical evidence. The theoretical analysis suggests that a source of the state dependency is retirees' lack of access to the labor market, due to which retirees cannot benefit from the increase in disposable income caused by a positive government spending shock. Therefore, our findings offer a possible implication that the efficacy of a government spending shock on economic activity could be improved by creating an environment in which elderly people could join the workforce easily. Given the limited scale of the fiscal stimulus package caused by an excess accumulation of sovereign debt due to the ongoing corona pandemic, coupled with the continually aging the population in many developed countries, the findings of this study provide implications for the labor market structure to enhance the effectiveness of a government spending shock. However, future research could aim to determine the type of labor market system that would make elderly people's labor participation more flexible. This task is left to future work.

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A Appendix

A.1 Log-linearized model

Except for the fiscal variables in the text (i.e., $\hat{\tau}_t$, $\hat{g}_t$, and $\hat{b}_t$), we denote the log-deviation of a variable from its steady-state as $\hat{x}_t = \ln x_t - \ln x$. Then, we summarize the log-linearized equilibrium conditions as follows:

Marginal utility of consumption	$\hat{\lambda}_t^R = \Theta_c \hat{c}_{w,t}^R + \Theta_g \hat{g}_t$
Euler equation	$\hat{\lambda}_t^R = E_t \hat{\lambda}_{t+1}^R + \hat{r}_t - E_t \hat{\pi}_{t+1}$
Non-Ricardians' consumption	$\gamma_c \hat{c}_{w,t}^N = \hat{w}_t + \hat{h}_t - \hat{\tau}_t$
Aggregate consumption	$\hat{c}_t = (1-\xi)(1-\zeta)\hat{c}_{w,t}^R + \xi(1-\zeta)\hat{c}_{w,t}^N$
Wage schedule	$\hat{w}_t = -\dfrac{\Theta_c}{1-\zeta}\hat{c}_t - \Theta_g \hat{g}_t + \mu \hat{h}_t$
New Keynesian Phillips Curve	$\hat{\pi}_t = \dfrac{1}{1+\rho} E_t \hat{\pi}_{t+1} + \dfrac{(1-\theta)(1+\rho-\theta)}{\theta(1+\rho)} \hat{w}_t$
Government budget constraint	$\hat{b}_t = R(\hat{g}_t - \hat{\tau}_t + \hat{b}_{t-1}) + \gamma_b(\hat{r}_t - R\hat{\pi}_t)$
Tax rule	$\hat{\tau}_t = \phi_b \hat{b}_{t-1} + \phi_g \hat{g}_t$
Taylor rule	$\hat{r}_t = \phi_\pi \hat{\pi}_t$
Market clearing	$\hat{y}_t = \gamma_c \hat{c}_t + \hat{g}_t$
Production function	$\hat{y}_t = \hat{h}_t$
Government spending	$\hat{g}_t = \rho_g \hat{g}_t + u_t^g$

where $\gamma_c = c/y$, $\gamma_g = g/y$, $\gamma_b = b/y$, and the associated coefficients are given by

$$\Theta_c \equiv \left[\frac{1-\sigma\eta}{\Gamma\eta} \omega c^{\frac{\eta-1}{\eta}} - \frac{1}{\eta} \right], \tag{48}$$

$$\Theta_g \equiv \frac{1-\sigma\eta}{\Gamma\eta}(1-\omega) g^{-\frac{1}{\eta}} y, \tag{49}$$

$$\Gamma = \left(\omega c^{\frac{\eta-1}{\eta}} + (1-\omega) g^{\frac{\eta}{\eta-1}}\right). \tag{50}$$

A.2 Steady-state values

Given the steady-state value of labor supply $h = \bar{h}$, we immediately obtain

$$y = \bar{h}, \tag{51}$$

from the production function. Then, the steady-state values of consumption and government spending are

$$c = \gamma_c \bar{h}, \tag{52}$$

$$g = \gamma_g \bar{h}, \tag{53}$$

from the market-clearing condition.

On the other hand, the first-order condition of the Ricardians' problem yields the gross nominal interest rate:

$$R = 1 + \rho \tag{54}$$

in the zero-inflation steady state. Given the rate of time preference exogenously, we can determine the gross nominal interest rate.

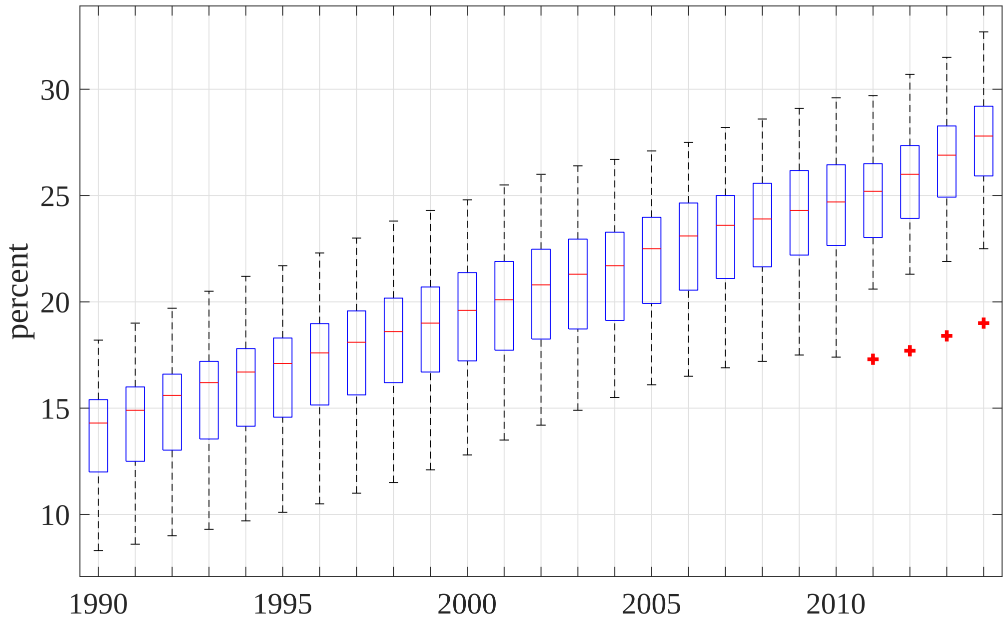

Figure 1: Time profile of the distribution of the aging rate among Japan's prefectures

Notes: This figure shows the distribution of the aging rate among Japan's prefectures for the period from 1990 to 2014 using a box plot. The tops and bottoms of each box are the 25th and 75th percentiles; the line in the box is the sample median; the lines extending from above and below each box are the ends of the interquartile ranges to the furthest observations that are less than 1.5 times the interquartile range from the top or bottom of the box; and outliers of more than 1.5 times the interquartile range away from the top or bottom of the box, are displayed with +.

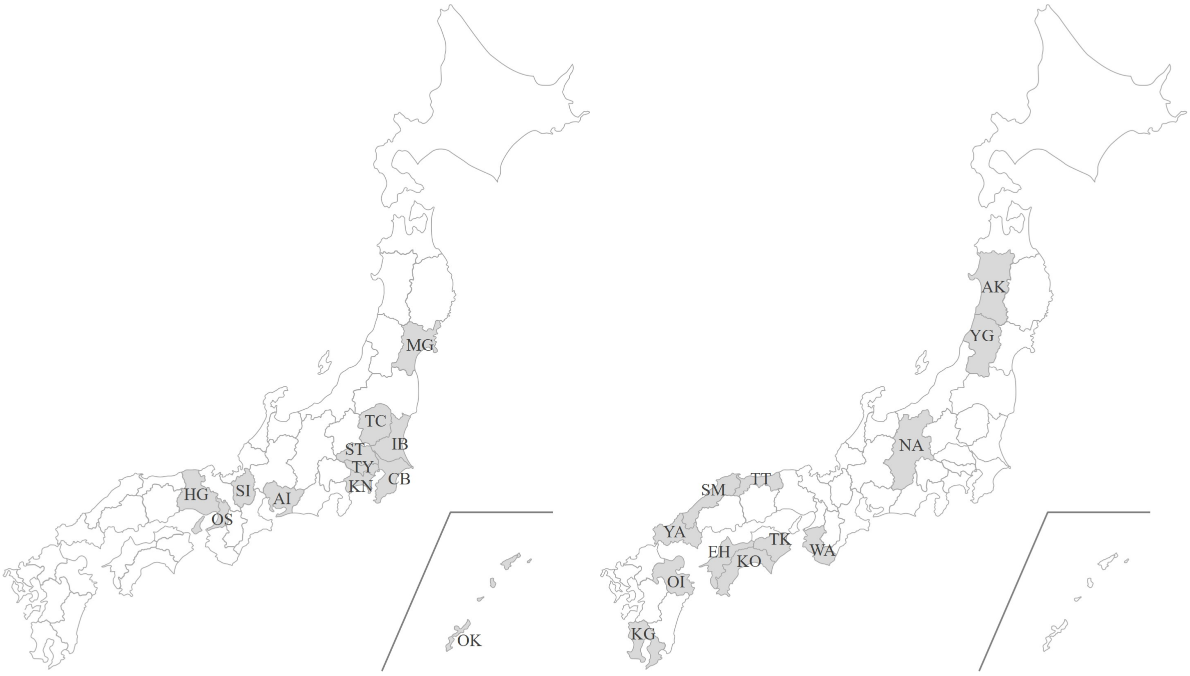

Figure 2:

Notes: This figures show the selected low and high aging prefectures, respectively. The low (high) aging prefectures are painted in the left-hand (right-hand) side of the map.

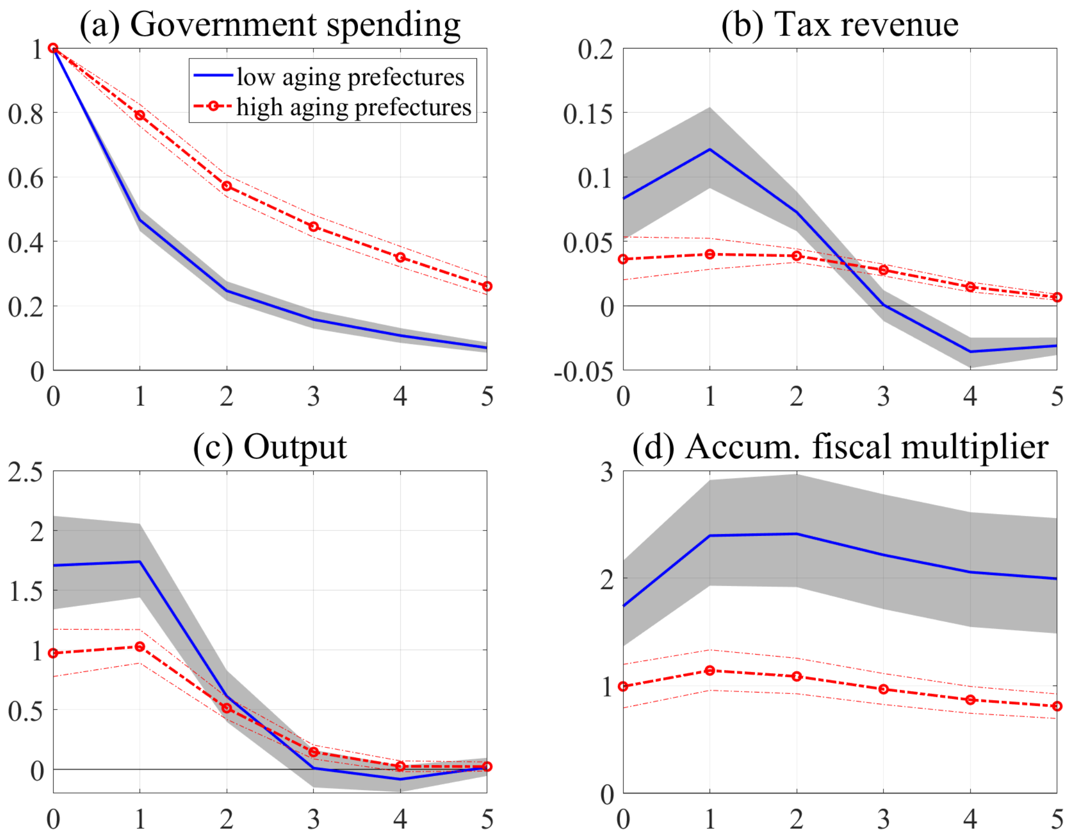

Figure 3: Empirical impulse responses to a government spending shock

Notes: This figure shows the empirical impulse responses of (a) government spending, (b) tax revenue, (c) output, and (d) the fiscal multipliers to a positive government spending shock for the two groups. The median responses obtained from the low- and high-aging prefectures are denoted by solid lines and dash-dotted lines with circles, respectively. In addition, the 68% credible intervals are depicted by the shaded areas and dash-dotted lines.

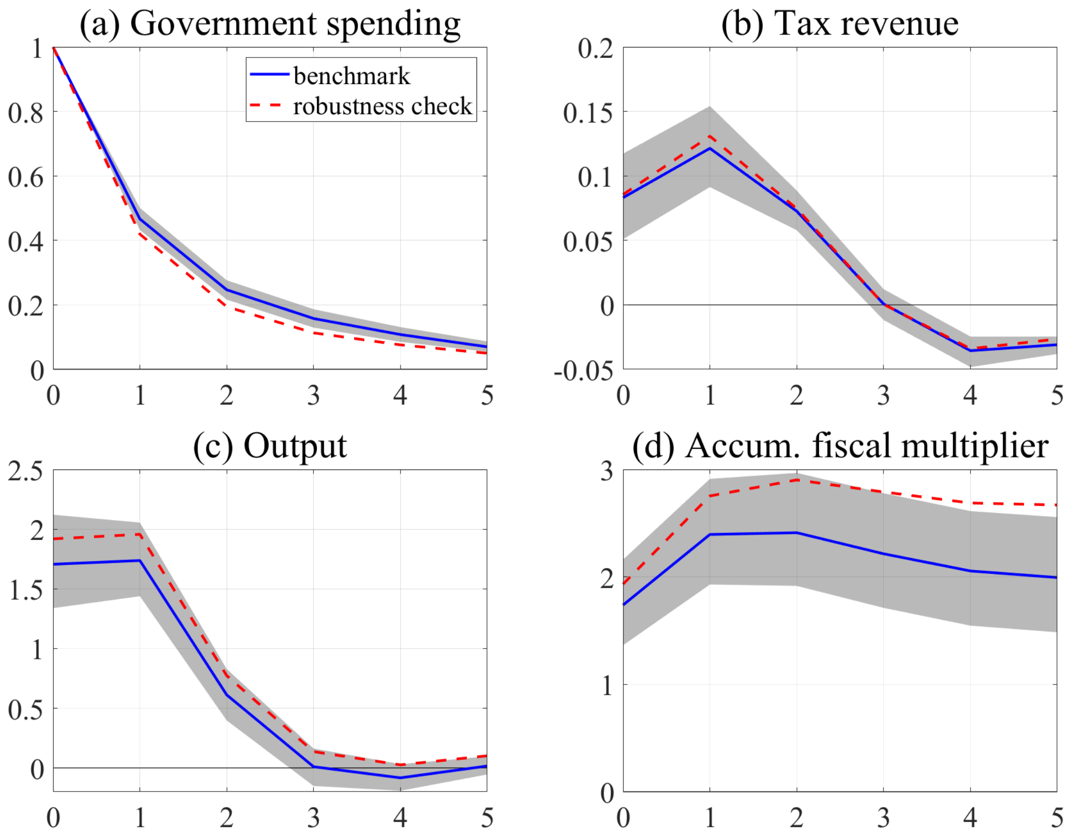

Figure 4: Empirical responses to a government spending shock (excluding the outlier, Okinawa)

Notes: This figure shows the empirical responses to a positive government spending shock in low-aging prefectures except for Okinawa prefecture.

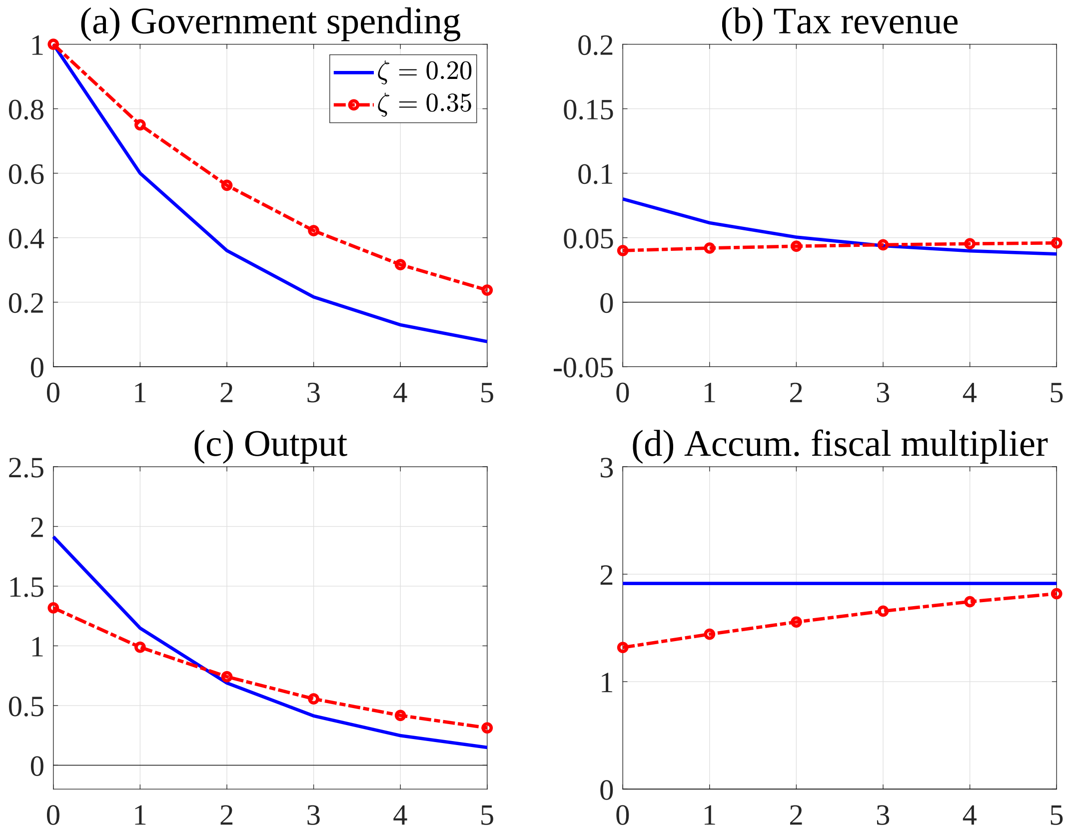

Figure 5: Theoretical responses to a government spending shock

Notes: This figure shows the theoretical responses of (a) government spending, (b) tax revenue, (c) output, and (d) the fiscal multiplier to a positive government spending shock. The responses in low- and high-aging economies are plotted by solid lines and dashed-dotted lines with circles, respectively, as with the empirical responses.

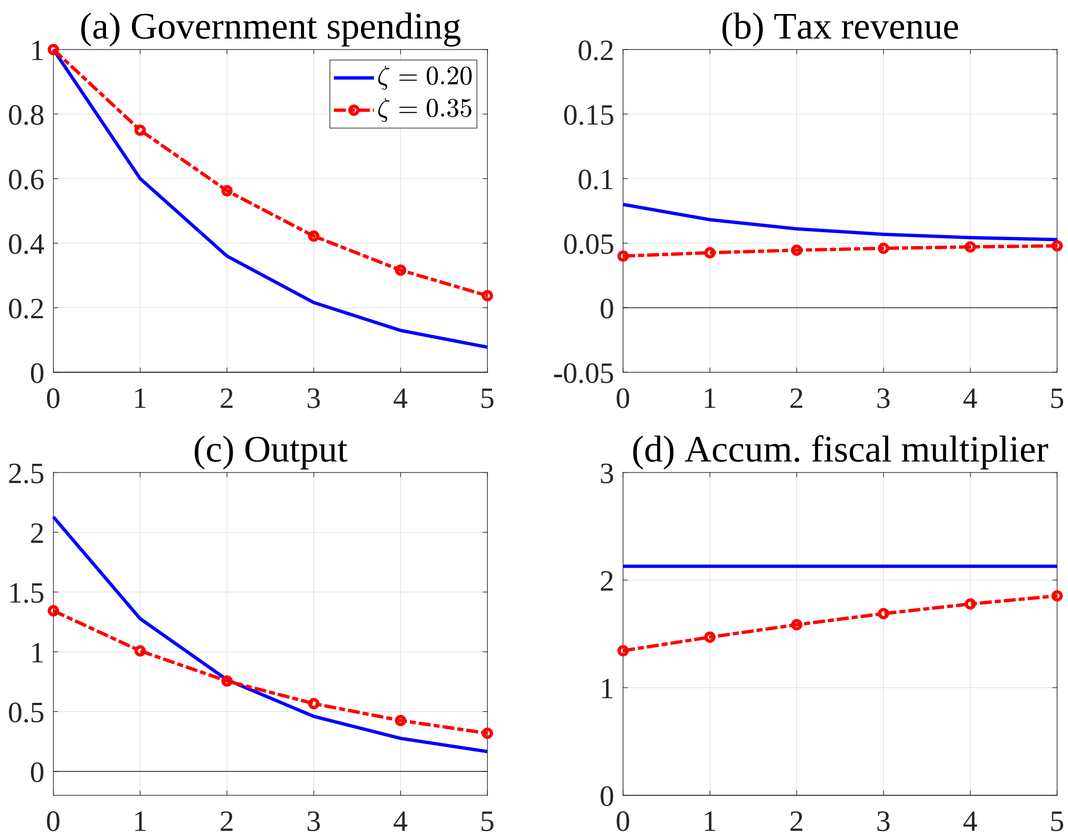

Figure 6: Theoretical responses to a government spending shock (including Ricardian retirees)

Notes: These figures show the theoretical responses derived from a model in which retirees can behave as Ricardians.

Table 1: Sign restrictions

	g	τ	x
Gov. spending	> 0		
Tax revenue		> 0	< 0
Business cycle	< 0	> 0	> 0

Notes: This table summarizes the sign restrictions imposed on the impulse response functions. The sign restrictions, if any, are imposed on each variable for two periods. A blank space indicates no restrictions.

Table 2: Calibration parameters

Parameters	Description	Value	
		Low	High
ρ	Time preference rate	0.01	
σ	Relative risk aversion	1.5	
μ	Inverse of labor supply elasticity	1.0	
ω	Weight attached to private consumption	0.8	
η	Elasticity of substitution	0.3	
θ	Calvo parameter	0.55	
λ	Fraction of Non-Ricardians	0.13	
ϕ_b	Tax elasticity of bonds	0.01	
ϕ_g	Tax elasticity of government spending	0.08	0.04
ρ_g	Persistence of government spending shock	0.60	0.75
ϕ_π	Taylor rule coefficients for inflation	1.2	
γ_c	Share of consumption in output	0.7	
γ_g	Share of government spending in output	0.3	
γ_b	Debt-to-output ratio	1.5	

Notes: This table summarizes the values of the calibrated parameters in our theoretical model based on the existing literature and our empirical results from the prior section. Note that several fiscal policy parameters change depending on the demographic stricture.