

# Inference in Weak Factor Models

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## Abstract

In this paper, we consider statistical inference for high-dimensional approximate factor models. We posit a weak factor structure, in which the factor loading matrix can be sparse and the signal eigenvalues may diverge more slowly than the cross-sectional dimension,  $N$ . We propose a novel inferential procedure to decide whether each component of the factor loadings is zero or not, and prove that this controls the false discovery rate (FDR) below a pre-assigned level, while the power tends to unity. This “factor selection” procedure is primarily based on a de-sparsified (or debiased) version of the WF-SOFAR estimator of [Uematsu and Yamagata \(2020\)](#), but is also applicable to the principal component (PC) estimator. After the factor selection, the *re-sparsified* WF-SOFAR and *sparsified* PC estimators are proposed and their consistency is established. Finite sample evidence supports the theoretical results. We apply our procedure to the FRED-MD macroeconomic and financial data, consisting of 128 series from June 1999 to May 2019. The results strongly suggest the existence of sparse factor loadings and exhibit a clear association of each of the extracted factors with a group of macroeconomic variables. In particular, we find a price factor, housing factor, output and income factor, and a money, credit and stock market factor.

**Keywords.** Approximate factor models, Debiased SOFAR estimator, Multiple testing, FDR and Power, Re-sparsification.

## 1 Introduction

The factor models have become an increasingly important tool for the analysis of psychology, finance, economics, and biology, among many others. This paper discusses statistical inference for high-dimensional *approximate factor models*. These were first introduced by [Chamberlain and Rothschild \(1983\)](#), then developed in subsequent articles by [Connor and Korajczyk \(1986, 1993\)](#), [Bai and Ng \(2002\)](#), [Bai \(2003\)](#), [Fan et al. \(2008\)](#), and [Fan et al. \(2011, 2013\)](#), among many others.

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