Inference in Weak Factor Models

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Abstract

In this paper, we consider statistical inference for high-dimensional approximate factor models. We posit a weak factor structure, in which the factor loading matrix can be sparse and the signal eigenvalues may diverge more slowly than the cross-sectional dimension, N. We propose a novel inferential procedure to decide whether each component of the factor loadings is zero or not, and prove that this controls the false discovery rate (FDR) below a pre-assigned level, while the power tends to unity. This "factor selection" procedure is primarily based on a de-sparsified (or debiased) version of the WF-SOFAR estimator of Uematsu and Yamagata (2020), but is also applicable to the principal component (PC) estimator. After the factor selection, the *re-sparsified* WF-SOFAR and *sparsified* PC estimators are proposed and their consistency is established. Finite sample evidence supports the theoretical results. We apply our procedure to the FRED-MD macroeconomic and financial data, consisting of 128 series from June 1999 to May 2019. The results strongly suggest the existence of sparse factor loadings and exhibit a clear association of each of the extracted factors with a group of macroeconomic variables. In particular, we find a price factor, housing factor, output and income factor, and a money, credit and stock market factor.

Keywords. Approximate factor models, Debiased SOFAR estimator, Multiple testing, FDR and Power, Re-sparsification.

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